

Elasticity, revenue and who bears a tax

Measure proportional responses, connect demand elasticity to revenue, solve a tax-wedge example and separate causal responses from observed correlations. All examples are illustrative.

Written walkthrough. Work through each explanation alongside the PDF slides. The lesson page also includes a captioned video with a generic synthetic narrator. It is not a recording of Craig Wright.

Fictional examples teach reasoning rather than describe a real business. No external academic credit is awarded.

Learning objectives

1. Calculate signed and absolute midpoint price elasticity with correct bases.
2. Distinguish elasticity from slope and interpret the revenue relationship.
3. Solve a basic per-unit tax model using separate buyer and seller prices.
4. Evaluate the assumptions behind elasticity estimates and causal claims.

Slide 1: Percentage changes make responses comparable

A change of ten units means different things in a market selling twenty units and one selling ten thousand. Percentages express a change relative to a base. Elasticity compares percentage changes in two variables. All data in this lesson are illustrative. Distinguish a percentage change in a rate from a percentage-point change.

Walkthrough

If a quantity rises from 50 to 55, the change is five units and the starting-base percentage increase is five divided by 50, or ten per cent. If an interest rate rises from four per cent to five per cent, it rises by one percentage point but by 25 per cent relative to its initial value. These are different measures. Elasticity uses proportional changes, allowing comparisons across units such as boxes, kilograms or litres. Always identify the relevant base and sign before calculating. A calculator cannot correct an ambiguous definition of the percentage you intend to measure.

Slide 2: Demand elasticity is a conditional response

Price elasticity of demand is the percentage change in quantity demanded divided by the percentage change in own price, along an unchanged demand relationship. Ordinary downward-sloping demand gives a negative signed elasticity. We usually classify its absolute magnitude: above one is elastic, below one is inelastic, and exactly one is unit elastic.

Walkthrough

An elasticity of minus two means that, locally, a one per cent price increase is associated with about a two per cent reduction in quantity demanded, other relevant conditions unchanged. The absolute magnitude is two. Be explicit about whether you report the signed value or its magnitude. Elastic does not mean that quantity is high; it means quantity responds proportionally more than price. An inelastic response is not necessarily zero. Perfectly inelastic demand has zero quantity response in the specified model and range, a much stronger condition than merely having a magnitude below one.

Slide 3: Worked example: use the midpoint method

Price rises from £10 to £12 and quantity falls from 100 to 80 units on one demand curve. Average price is £11; average quantity is 90. Midpoint changes are $2/11 = 18.18\%$ and $-20/90 = -22.22\%$. Signed elasticity is -1.222 approximately; its magnitude exceeds one, so this interval is elastic.

Walkthrough

Step one calculates the changes: price increases by two pounds and quantity decreases by twenty units. Step two calculates the average of each pair of endpoints: eleven pounds and ninety units. Step three divides each change by its own average, giving approximately 0.1818 and negative 0.2222. Step four divides the quantity proportion by the price proportion. The result is negative eleven-ninths, or approximately negative 1.222. Using midpoint bases gives the same magnitude when the direction of comparison is reversed. Do not divide a percentage quantity change by a raw pound change; that would not be an elasticity.

Slide 4: Revenue connects price and quantity

Total revenue equals price multiplied by quantity sold. In the preceding example, revenue changes from £1,000 to £960. The price rise is outweighed by the quantity response. Along unchanged demand, an elastic interval makes price and revenue move in opposite directions; an inelastic interval makes them move in the same direction.

Walkthrough

Calculate revenue independently to check the interpretation: ten pounds times one hundred is one thousand pounds; twelve times eighty is nine hundred and sixty. Revenue falls by forty pounds. For positive endpoints, the midpoint elasticity classification is consistent with this finite-change revenue test. If its magnitude is exactly one, the two endpoint revenues are equal. The relationship assumes the observed quantity change is a movement along unchanged demand and that all units sell at the stated price. Revenue is not profit. Costs, returns, discounts and capacity constraints may change whether a pricing decision benefits the seller.

Slide 5: Worked example: an inelastic price cut

Price falls from £20 to £18 while quantity rises from 50 to 54. Average price is £19 and average quantity is 52. The midpoint elasticity is $(4/52)$ divided by $(-2/19)$, approximately -0.731 . Revenue falls from £1,000 to £972: the percentage quantity gain is too small to offset the price reduction.

Walkthrough

First take the signed changes: quantity rises by four and price falls by two. Divide by the midpoint bases to get about positive 7.692 per cent and negative 10.526 per cent. Dividing those proportions gives negative nineteen-twenty-sixths, approximately negative 0.731. Its magnitude is below one, so demand over this interval is inelastic. Next calculate revenues: 20 times 50 equals 1,000, while 18 times 54 equals 972. The price cut reduces revenue by £28. More customers or more units sold therefore does not necessarily imply more sales revenue, still less more profit.

Slide 6: A straight line can have changing elasticity

Slope compares changes in units; elasticity compares proportional changes. For $Q = 200 - 10P$, the slope is always minus ten units per pound. Point elasticity is $-10P/Q$. At $P = £5$ and $Q = 150$ its magnitude is one-third; at $P = £15$ and $Q = 50$ it is three.

Walkthrough

The same ten-unit response is a larger percentage of a small starting quantity than of a large one. Likewise, a one-pound change is a smaller percentage of a high price than of a low price. That is why a constant slope need not imply constant elasticity. The point formula multiplies the quantity response per pound by price divided by quantity. At five pounds, it gives negative ten times five divided by 150. At fifteen pounds, it gives negative ten times fifteen divided by fifty. Point elasticity describes a small local change; midpoint elasticity summarises an interval between two distinct observations.

Slide 7: Substitution and time influence responsiveness

Demand often responds more when close substitutes are available, when spending represents a larger budget share, or when buyers have time to adjust. These are tendencies to investigate, not universal numerical laws. A narrowly defined product can face more substitutes than a broad category. Essential uses do not make every quantity completely unresponsive.

Walkthrough

A particular café may have many nearby substitutes, while coffee as a broad category has fewer close alternatives for some customers. Buyers may respond to a lasting fuel-price change by changing routes, vehicles or housing, even if tomorrow's commute barely changes. The relevant time horizon therefore matters. Different groups can also have different responses. An average market elasticity can conceal people who have few short-run alternatives. To use an estimate responsibly, check its product definition, population, period, price range and research design. An elasticity borrowed from a different setting may be a poor guide.

Slide 8: Supply has an elasticity too

Price elasticity of supply compares the percentage change in quantity supplied with the percentage change in the price received by sellers, other supply conditions unchanged. It is often positive. Capacity, inventories, input availability and time to expand matter. Demand responsiveness and supply responsiveness jointly determine how a market adjusts to a disturbance.

Walkthrough

A bakery with unused ovens may increase output quickly, while a fully booked venue cannot create more seats for tonight merely because offers rise. Over a longer period, investment or entry can make supply more responsive. These examples concern conditional responses, not shifts caused by new technology or input-price changes. As with demand, distinguish a local elasticity from an estimate over a large interval. A vertical supply curve means fixed quantity in the model's relevant range, whereas a horizontal curve represents an extremely responsive supply at the given price. Neither shape is appropriate for every market.

Slide 9: Other elasticities answer other questions

Income elasticity relates quantity demanded to income. A positive value describes a normal good; a negative value describes an inferior good over the relevant range. Cross-price elasticity relates demand for one good to another good's price. Positive responses can indicate substitutes; negative responses can indicate complements, with relevant conditions held fixed.

Walkthrough

Do not interpret all elasticities as own-price demand elasticity. If bus fares rise and train demand increases, the cross-price response can indicate substitution between travel options. If printer prices rise and demand for ink falls, the goods may be complements. These labels depend on the context and level of aggregation; they are not timeless characteristics of every individual's preferences. Income elasticity also varies across groups and income ranges. The word inferior has no moral meaning here. As always, an observed correlation is not automatically a causal response when incomes, prices and other conditions move together.

Slide 10: A tax introduces a price wedge

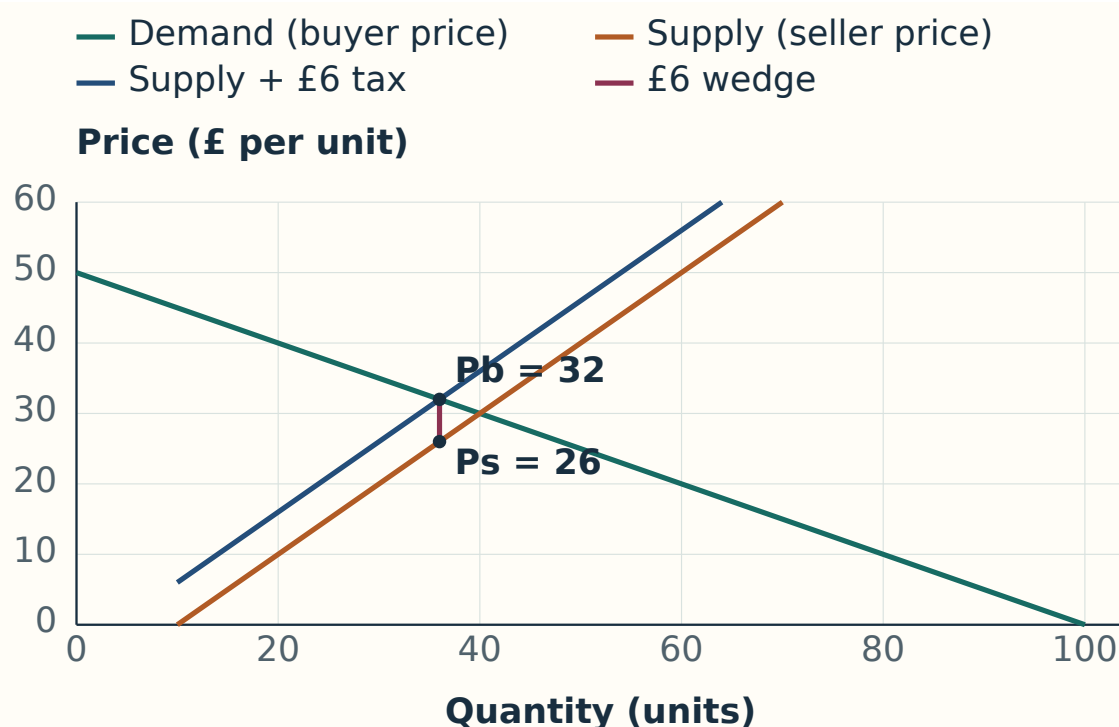
For a per-unit tax, distinguish the price buyers pay, P_b , from the net price sellers receive, P_s . With a £6 tax, $P_b = P_s + 6$. Our illustrative schedules are $Q_d = 100 - 2P_b$ and $Q_s = 10 + P_s$. Without the tax, one common price is £30 and quantity is forty.

Walkthrough

Before introducing the tax, set buyer and seller prices equal to P . Equilibrium is $100 \text{ minus } 2P \text{ equals } 10 \text{ plus } P$, giving $90 \text{ equals } 3P$ and $P \text{ equals } 30$. Supply then gives 40 units. With the tax, using a single price in both original schedules would erase the distinction we need to analyse. Buyers respond to the amount they pay, while sellers respond to what they retain per unit after the tax. We assume a competitive market, compliance, unchanged underlying schedules and no separate administrative costs or other policy effects.

Slide 11: Worked example: calculate tax incidence

Substitute $P_b = P_s + 6$ into demand: $100 - 2(P_s + 6) = 10 + P_s$. This simplifies to $78 = 3P_s$, so $P_s = £26$ and $P_b = £32$. Quantity is thirty-six. Compared with the untaxed price, buyers pay £2 more and sellers receive £4 less per unit.



At 36 units buyers pay £32 and sellers receive £26; the vertical wedge is £6.

Walkthrough

Expand the left side carefully: $100 \text{ minus } 2P_s \text{ minus } 12 \text{ equals } 10 \text{ plus } P_s$. Subtract ten and collect price terms to obtain $78 \text{ equals } 3P_s$. Dividing by three gives 26, then adding the six-pound tax gives the buyer price of 32. Check quantities: demand is $100 \text{ minus } 64$, or 36; supply is $10 \text{ plus } 26$, also 36. Tax revenue is six times 36, or £216. The buyer and seller burdens per traded unit sum to the tax: two plus four equals six. This is a particular model calculation, not a general two-to-one incidence rule.

Slide 12: Legal payment and economic burden can differ

Who sends a tax payment to the authority need not determine who bears its economic cost. In the simple competitive model, the less responsive side generally bears more of a small tax's price burden. At the untaxed equilibrium here, supply elasticity is 0.75 and demand's magnitude is 1.5, consistent with sellers bearing more.

Walkthrough

Use the local elasticity formulas at the original price of 30 and quantity of 40. Supply's slope is one, so its elasticity is one times 30 divided by 40, or 0.75. Demand's slope is negative two, giving magnitude two times 30 divided by 40, or 1.5. Sellers are less responsive in this example. This intuition concerns relative ability to adjust quantities. Market power, evasion, price regulation, cross-border options and longer-run changes can alter the result. A positive incidence calculation also does not decide whether a tax is justified; that judgement needs stated goals, rights and other consequences.

Slide 13: A correlation is not a demand experiment

A hot weekend can increase both cold-drink prices and sales. That does not establish an upward-sloping demand curve; demand may have shifted. To estimate a causal price response, isolate a price change from other influences. Randomised prices or credible quasi-experimental variation can help, but each design needs assumptions, appropriate permissions and careful interpretation.

Walkthrough

Imagine comparing a quiet rainy Tuesday with a crowded sunny Saturday. Weather, visitors, opening hours and stock could differ alongside price. Dividing the observed percentage quantity change by the observed price change gives a descriptive ratio, but not necessarily demand elasticity. A controlled comparison might hold other conditions more nearly constant, while a research design may use a source of price variation unrelated to demand changes. Neither method is automatically flawless. Check selection, spillovers, measurement, compliance and the range of prices studied. Report uncertainty rather than presenting a single estimate as a universal behavioural constant.

Slide 14: Calculate, interpret, then qualify

Name the response being measured. Use consistent percentage bases and report whether elasticity is signed or absolute. Check revenue directly when relevant, and separate revenue from profit. For incidence, distinguish buyer and seller prices. Finally, ask whether other conditions were genuinely controlled and whether the estimate applies to the decision at hand.

Walkthrough

A reliable answer has three layers. First comes arithmetic: changes, averages, ratios and units. Second comes interpretation: what the magnitude means for the specified demand or supply relationship. Third comes identification and scope: why the data support a causal statement and where the estimate might fail to transfer. The exercises deliberately include a case where arithmetic is possible but a causal demand interpretation is not established. Written reflections remain ungraded. Use them to explain assumptions and evidence, because checking a number cannot substitute for understanding the model or the way the observations were generated.

Questions students ask

Should I report a negative elasticity or a positive one?

State the convention. Signed own-price demand elasticity is normally negative for downward-sloping demand. Elastic, inelastic and unit-elastic classifications commonly use its absolute magnitude.

Can a price cut increase sales and reduce revenue?

Yes. With inelastic demand over the interval, the proportional quantity increase is too small to offset the proportional price reduction. Calculate price times quantity at both endpoints to check.

Can the party legally paying a tax pass all of it on?

That is not automatic. In the simple competitive model, the division depends on demand and supply responsiveness. Other market institutions and adjustment mechanisms can also matter.

Your exercises

Submit your answers on the lesson page at craigwright.net/learning. Numerical and choice checks receive server feedback. Written responses are saved ungraded. No assessment answer key appears in this guide.

1. Calculate an interval response

Illustrative unchanged demand curve: price rises from £8 to £10 and quantity falls from 150 to 120 units. Calculate the absolute midpoint elasticity and total revenue at each endpoint. Round elasticity to two decimal places if needed.

Absolute midpoint elasticity

Revenue before (£)

Revenue after (£)

2. Evaluate another price cut

Illustrative unchanged demand curve: price falls from £30 to £27 and quantity rises from 200 to 210. Calculate absolute midpoint elasticity to two decimal places, the signed change in revenue (after minus before), and the elasticity classification.

Absolute midpoint elasticity

Revenue after minus before (£)

Classification

1. Elastic
2. Inelastic
3. Unit elastic

3. Solve a new tax wedge

Illustrative competitive market: $Q_d = 120 - 3P_b$ and $Q_s = 12 + P_s$. A per-unit tax of £8 creates $P_b = P_s + 8$. Calculate the buyer price, seller net price and quantity after the tax. All schedules remain unchanged apart from this wedge.

Buyer price after tax (£ per unit)

Seller net price after tax (£ per unit)

Quantity after tax (units)

Ungraded reflection: what would you compare to calculate each side's burden per traded unit?

4. Separate a ratio from a causal estimate

Illustrative café observations: on a quiet weekday, price is £4 and sales are 80 drinks; during a busy festival, price is £5 and sales are 100. Calculate the signed midpoint percentage-change ratio (quantity change divided by price change). Then decide what the observations alone establish about causal own-price demand elasticity.

Signed midpoint ratio

Supported conclusion

1. Causal demand elasticity must be positive
2. Causal demand elasticity must be negative one
3. Causal demand elasticity is not identified by these observations alone

Ungraded reflection: name one plausible confounding change.