

Letters stand for quantities

A variable is a symbol whose value can change or is not yet known. A constant is held fixed within the problem. In $C = 18 + 6q$, q represents quantity and C represents total cost in pounds. The fixed charge is £18 and the cost per unit is £6. Writing $6q$ means $6 \times q$.

Expressions, equations and solutions

An expression, such as $3x + 7$, describes a calculation. An equation, such as $3x + 7 = 22$, states that two expressions have equal values. A solution is a value that makes the equation true. A linear equation contains variables only to the first power, with no products between variables.

Brackets and the order of operations

Calculate brackets first, then powers, then multiplication and division, then addition and subtraction. Operations at the same level are evaluated from left to right. Distributing multiplication gives $3(x + 2) = 3x + 6$. Collect like terms only: $4x + 2x = 6x$, but $4x + 2$ cannot be combined into $6x$.

Substitution turns a formula into arithmetic

Substitution means replacing a symbol with its stated value. For $C = 18 + 6q$ and $q = 4$, calculate $C = 18 + 6 \times 4 = 18 + 24 = \text{£}42$. Multiply before adding. Check units: pounds plus pounds per unit multiplied by units gives pounds, as required for total cost.

Preserve the balance of an equation

To solve an equation, use operations that preserve equality. Add or subtract the same quantity on both sides; multiply or divide both sides by the same non-zero number. For $3x + 7 = 22$, subtract 7: $3x = 15$. Divide by 3: $x = 5$. Check: $3 \times 5 + 7 = 22$.

Unknowns can appear on both sides

For $5x + 7 = 2x + 22$, subtract $2x$ from both sides: $3x + 7 = 22$. Subtract 7: $3x = 15$. Divide by 3: $x = 5$. Check both original sides: $5 \times 5 + 7 = 32$ and $2 \times 5 + 22 = 32$.

Inequalities describe ranges of answers

An inequality compares sizes: $<$ means less than, $\leq$ means less than or equal to, $>$ means greater than, and $\geq$ means greater than or equal to. Adding the same quantity preserves the sign. Multiplying or dividing by a negative number reverses it: $-2x < 6$ becomes $x > -3$ after division by -2 .

Worked example: turn a cost question into an equation

A fictional workshop charges £18 for setup and £6 per unit. A total bill is £90. Let q be the number of units, so $18 + 6q = 90$. Subtract 18 from both sides: $6q = 90 - 18 = 72$. The £72 remaining covers only the per-unit charges.

Worked example: solve, check and interpret

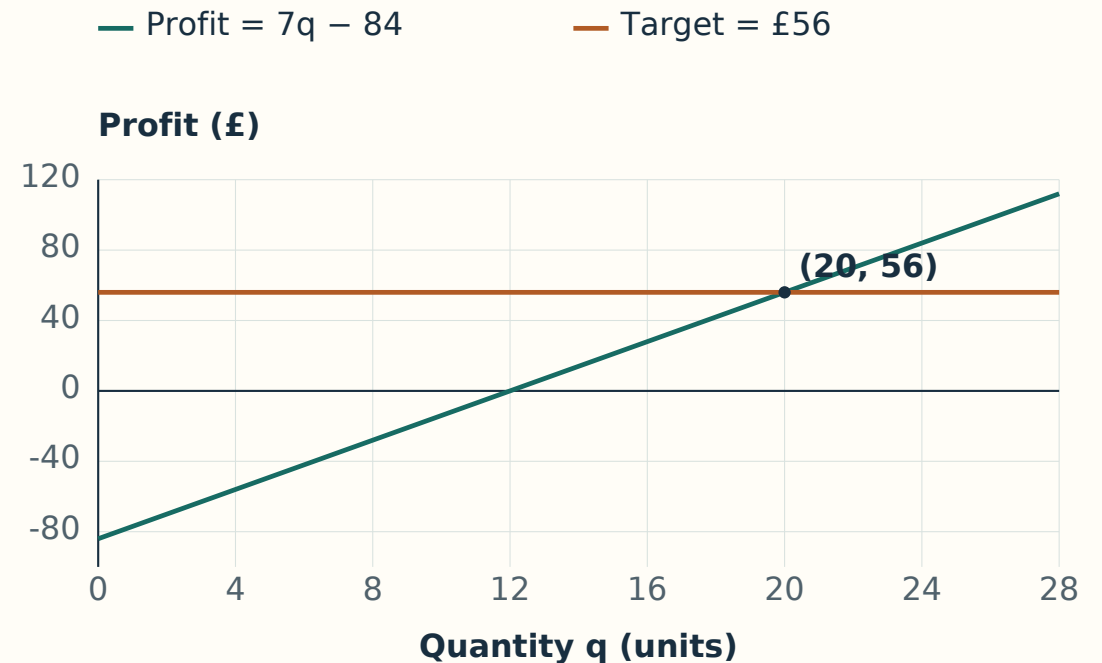
From $6q = 72$, divide both sides by 6: $q = 72 \div 6 = 12$ units. Check the original bill: $\text{£}18 + \text{£}6 \times 12 = \text{£}18 + \text{£}72 = \text{£}90$. The result is a whole, non-negative quantity, so it fits this workshop model as well as satisfying the equation.

Worked example: model revenue, cost and profit

An invented business receives £18 per unit sold, pays £11 per unit in variable costs and has £84 in fixed costs. Revenue $R = 18q$; cost $C = 84 + 11q$. Profit π means revenue minus cost: $\pi = 18q - (84 + 11q) = 18q - 84 - 11q = 7q - 84$.

Worked example: meet a minimum profit target

Require profit of at least £56: $7q - 84 \geq 56$. Add 84 to both sides: $7q \geq 140$. Divide by positive 7: $q \geq 20$.
Check $q = 20$: revenue = £360, cost = £84 + £220 = £304, profit = £56.
At least 20 whole units are required under the model.



Rearrange formulas before inserting numbers

Rearranging makes a chosen variable the subject, meaning it stands alone on one side. Starting from $C = F + vq$, subtract F : $C - F = vq$. Divide by v : $q = (C - F)/v$, provided $v \neq 0$. To isolate v instead, divide by q : $v = (C - F)/q$, provided $q \neq 0$.

Mathematical answers need domain checks

The domain is the set of values allowed for a variable. Counts of indivisible products are usually non-negative whole numbers; hours or mass may be fractional. An inequality such as $q \geq 6.2$ requires at least 7 whole products. Equations can also have no solution or many solutions, so do not force every problem into one numerical answer.

An algebra routine you can explain

Define each symbol and its units. Translate the words into an equation or inequality. Simplify carefully, apply the same valid operation to both sides, and reverse inequality signs when dividing or multiplying by negatives. Substitute the result into the original relationship. Finally, apply whole-number, non-negativity or other restrictions required by the scenario.