

A number needs a meaning

A quantity combines a number with a meaning: £24 is money, 24 kilograms is mass, and 24 hours is time. A unit is the standard used to measure that quantity. In this lesson, every answer should say what is being counted or measured. Our numerical scenarios are invented for learning.

Totals and rates answer different questions

A total describes an amount, such as £48 spent. A rate compares quantities with different units, such as £12 per hour. The word “per” means “for each” and usually signals division. Multiplying a rate by the relevant quantity produces a total: $(£12 \text{ per hour}) \times 4 \text{ hours} = £48$.

Convert units before combining quantities

A conversion changes the numerical description, not the physical amount. One kilogram equals 1,000 grams, so $300 \text{ grams} = 300 \div 1,000 = 0.3$ kilograms. One hour equals 60 minutes. Before adding quantities or comparing prices, express the relevant measurements in matching units. Never add kilograms directly to grams.

Fractions describe equal parts

A fraction a/b means a equal parts when the whole has been divided into b equal parts. The numerator a counts the parts; the denominator b sets their size. For $3/4$ of a £20 amount, divide £20 into four equal parts, then take three: $£20 \div 4 \times 3 = £15$.

Equivalent fractions and decimal notation

Equivalent fractions name the same share. Multiplying or dividing both numerator and denominator by the same non-zero number preserves the value: $6/8 = 3/4$. To write a fraction as a decimal, divide the numerator by the denominator: $3 \div 4 = 0.75$. A terminating decimal ends; some decimal representations continue indefinitely.

Calculate with fractions carefully

To add fractions, first use a common denominator: $1/2 + 1/3 = 3/6 + 2/6 = 5/6$. To multiply, multiply numerators and denominators: $(2/3) \times (3/5) = 6/15 = 2/5$. To divide by a non-zero fraction, multiply by its reciprocal, which swaps its numerator and denominator.

Ratios compare; rates include units

A ratio compares relative amounts. A red-to-blue ratio of 2:3 means two red items for every three blue items. There are five parts altogether, so red items form $\frac{2}{5}$ of the total. A unit rate gives an amount for one unit, such as pounds per kilogram, and helps compare differently sized purchases.

Worked example: plan a unit-price calculation

An invented flour pack contains 750 grams and costs £1.80. We want its price per kilogram and the cost of 4.5 kilograms at the same rate. First convert the mass: $750 \div 1,000 = 0.75$ kilograms. Then set up the rate: price per kilogram = $\text{£}1.80 \div 0.75$ kilograms.

Worked example: calculate and verify the cost

Price per kilogram = $1.80 \div 0.75 = 180 \div 75 = \text{£}2.40$ per kilogram. Cost of 4.5 kilograms = $2.40 \times 4.5 = 2.40 \times (4 + 0.5) = 9.60 + 1.20 = \text{£}10.80$.

Check another way: $4.5 \div 0.75 = 6$ packs; $6 \times \text{£}1.80 = \text{£}10.80$.

Worked example: divide a budget in a ratio

An illustrative £360 budget is divided between rent, food and travel in the ratio 5:3:1. The order matters: rent receives five parts, food three and travel one. Total parts = $5 + 3 + 1 = 9$. Value of one part = $£360 \div 9 = £40$.

Worked example: allocate and check the whole

Rent = $5 \times £40 = £200$. Food = $3 \times £40 = £120$. Travel = $1 \times £40 = £40$.
Check the total: $£200 + £120 + £40 = £360$. Food's fraction of the whole is $3/9 = 1/3$; this agrees with $£120 \div £360 = 1/3$.

Direct proportion: scale the same relationship

Two quantities are directly proportional when multiplying one by a factor multiplies the other by the same factor. If four identical tickets cost £18 with no fixed fee, one costs $£18 \div 4 = £4.50$. Seven therefore cost $7 \times £4.50 = £31.50$. The constant unit rate is the key assumption.

Use estimates and sensible precision

An estimate is a deliberately approximate calculation used to check size. Rounding £2.98 to about £3 makes ten items cost about £30. Precision should fit the question: ordinary money totals are usually stated to two decimal places. Keep intermediate values unrounded where possible, and state when an answer is approximate.

Your quantity-checking routine

Use five steps: identify the requested quantity; write the starting units; convert to compatible units; calculate; then check the result against the original situation. Ask whether a fraction refers to the right whole and whether a ratio keeps its stated order. A complete answer combines a value, a unit and a short interpretation.

Percentage means parts per hundred

A percentage expresses a quantity relative to one hundred equal parts. Thus $25\% = 25/100 = 0.25$. To convert a percentage to a decimal, divide by 100; to convert a decimal to a percentage, multiply by 100. A percentage alone does not identify the size of the underlying whole.

Find a percentage of a quantity

To find $p\%$ of an amount A , calculate $A \times p/100$. For example, 12% of £250
 $= 250 \times 12/100 = 250 \times 0.12 = £30$. Here £250 is the base: the whole
against which the percentage is measured. Label the base before doing the
multiplication.

Multipliers include the amount that remains

A multiplier is the factor used to obtain a new value. Increasing A by $p\%$ gives $A \times (1 + p/100)$. Decreasing A by $p\%$ gives $A \times (1 - p/100)$. A 20% increase uses 1.20; a 20% decrease uses 0.80. Multiplying by 0.20 finds only the change amount.

Percentage change uses the starting value

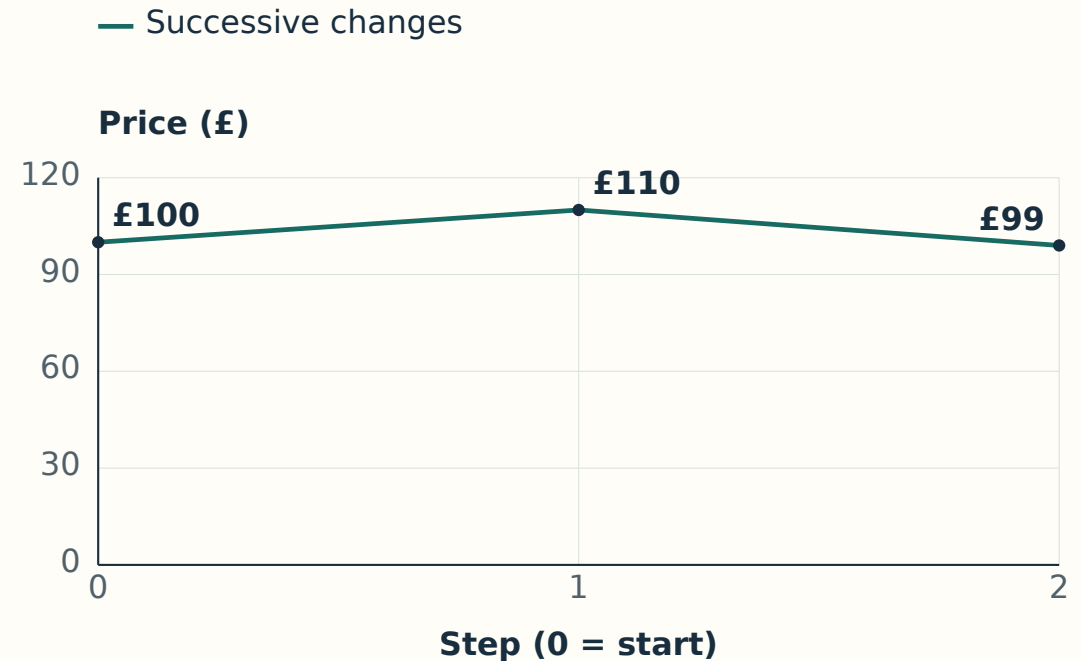
Percentage change = $[(\text{new value} - \text{original value}) \div \text{original value}] \times 100$.
If an illustrative quantity rises from 40 to 50, the change is 10 and the percentage change is $10 \div 40 \times 100 = 25\%$. A negative result indicates a decrease. The usual formula is undefined when the original value is zero.

Percentage points compare percentage levels

A percentage point is one unit on a percentage scale. If a fictional participation rate rises from 30% to 36%, it rises by $36 - 30 = 6$ percentage points. Relative to its original level, it rises by $(36 - 30) \div 30 \times 100 = 20\%$. These are different descriptions of the same movement.

Opposite percentages do not usually cancel

An invented price of £100 rises by 10% to £110. A subsequent 10% decrease removes £11, leaving £99. The second percentage uses a different base. The combined multiplier is $1.10 \times 0.90 = 0.99$, so the net movement is a 1% decrease, even though the stated percentages have equal sizes.



Compounding builds on earlier changes

Compounding means applying each percentage change to the amount reached after the previous change. At a constant decimal rate r for n periods, final amount = initial amount $\times (1 + r)^n$. The exponent n means repeated multiplication. With varying rates, multiply the separate period multipliers in sequence.

Worked example: discount an illustrative price

A fictional item lists at £120. A 15% discount is applied first, followed by a 5% charge on the discounted price. Discount amount = $£120 \times 15/100 = £18$. Discounted price = $£120 - £18 = £102$. Equivalently, $£120 \times 0.85 = £102$. The next calculation must use £102 as its base.

Worked example: apply the charge and compare

Charge = $£102 \times 0.05 = £5.10$. Final price = $£102 + £5.10 = £107.10$.

Combined multiplier = $0.85 \times 1.05 = 0.8925$. Net change = $(107.10 - 120) \div 120 \times 100 = -12.90 \div 120 \times 100 = -10.75\%$. The final price is 10.75% below the original price.

Worked example: set up repeated growth

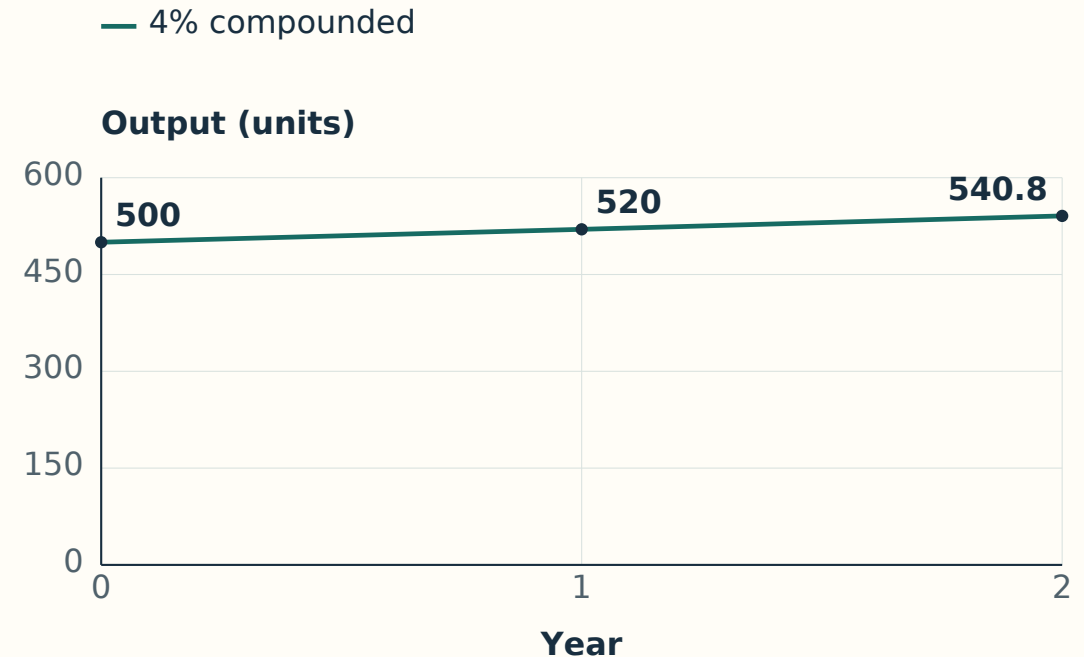
An invented planning model starts with 500 units of output. Output grows by 4% in each of two years, with each increase based on the previous year's output. The annual multiplier is $1 + 4/100 = 1.04$. After year one: $500 \times 1.04 = 520$ units. The second year starts from 520.

Worked example: finish and measure total growth

Year-two increase = $520 \times 0.04 = 20.8$ units. Final output = $520 + 20.8 = 540.8$ units. Equivalently, $500 \times 1.04^2 = 500 \times 1.0816 = 540.8$.

Overall percentage growth = $(540.8 - 500) \div 500 \times 100 = 8.16\%$.

Adding $4\% + 4\%$ would miss growth on the earlier increase.



Reverse a percentage change by dividing

If a value after a 15% increase is £69, the original value satisfies $\text{original} \times 1.15 = 69$. Therefore $\text{original} = 69 \div 1.15 = \text{£}60$. Subtracting 15% of £69 would use the wrong base. For a known discount, divide the final value by the fraction of the original that remains.

Report the base, period and rounding

A useful percentage statement identifies its base, its time period and whether the change is relative or measured in percentage points. Round only the final answer unless instructed otherwise. A large percentage increase can describe a small absolute change, so report the underlying amounts when they matter. Percentages alone cannot establish why a change occurred.

A reliable percentage decision tree

Ask what the question wants. A share requires $\text{base} \times \text{decimal percentage}$. A new amount requires a multiplier. A percentage change requires $\text{change} \div \text{original} \times 100$. A change between percentage levels may require percentage points. Successive changes require multiplied factors. Recovering an original value requires dividing by the known multiplier.

Letters stand for quantities

A variable is a symbol whose value can change or is not yet known. A constant is held fixed within the problem. In $C = 18 + 6q$, q represents quantity and C represents total cost in pounds. The fixed charge is £18 and the cost per unit is £6. Writing $6q$ means $6 \times q$.

Expressions, equations and solutions

An expression, such as $3x + 7$, describes a calculation. An equation, such as $3x + 7 = 22$, states that two expressions have equal values. A solution is a value that makes the equation true. A linear equation contains variables only to the first power, with no products between variables.

Brackets and the order of operations

Calculate brackets first, then powers, then multiplication and division, then addition and subtraction. Operations at the same level are evaluated from left to right. Distributing multiplication gives $3(x + 2) = 3x + 6$. Collect like terms only: $4x + 2x = 6x$, but $4x + 2$ cannot be combined into $6x$.

Substitution turns a formula into arithmetic

Substitution means replacing a symbol with its stated value. For $C = 18 + 6q$ and $q = 4$, calculate $C = 18 + 6 \times 4 = 18 + 24 = \text{£}42$. Multiply before adding. Check units: pounds plus pounds per unit multiplied by units gives pounds, as required for total cost.

Preserve the balance of an equation

To solve an equation, use operations that preserve equality. Add or subtract the same quantity on both sides; multiply or divide both sides by the same non-zero number. For $3x + 7 = 22$, subtract 7: $3x = 15$. Divide by 3: $x = 5$. Check: $3 \times 5 + 7 = 22$.

Unknowns can appear on both sides

For $5x + 7 = 2x + 22$, subtract $2x$ from both sides: $3x + 7 = 22$. Subtract 7: $3x = 15$. Divide by 3: $x = 5$. Check both original sides: $5 \times 5 + 7 = 32$ and $2 \times 5 + 22 = 32$.

Inequalities describe ranges of answers

An inequality compares sizes: $<$ means less than, $\leq$ means less than or equal to, $>$ means greater than, and $\geq$ means greater than or equal to. Adding the same quantity preserves the sign. Multiplying or dividing by a negative number reverses it: $-2x < 6$ becomes $x > -3$ after division by -2 .

Worked example: turn a cost question into an equation

A fictional workshop charges £18 for setup and £6 per unit. A total bill is £90. Let q be the number of units, so $18 + 6q = 90$. Subtract 18 from both sides: $6q = 90 - 18 = 72$. The £72 remaining covers only the per-unit charges.

Worked example: solve, check and interpret

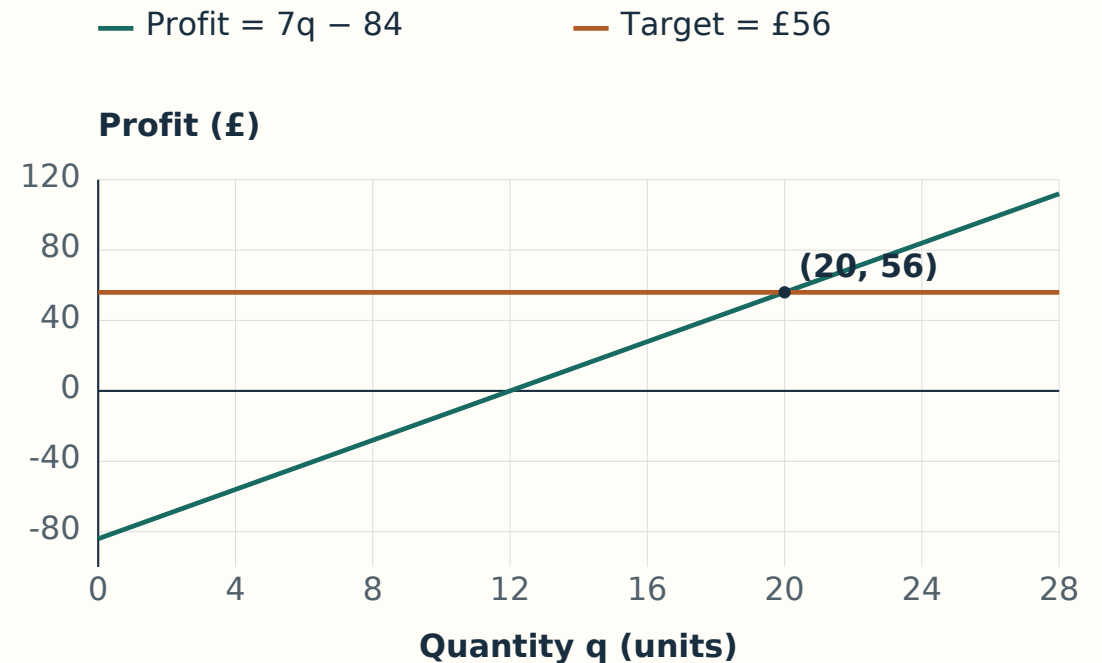
From $6q = 72$, divide both sides by 6: $q = 72 \div 6 = 12$ units. Check the original bill: $\text{£}18 + \text{£}6 \times 12 = \text{£}18 + \text{£}72 = \text{£}90$. The result is a whole, non-negative quantity, so it fits this workshop model as well as satisfying the equation.

Worked example: model revenue, cost and profit

An invented business receives £18 per unit sold, pays £11 per unit in variable costs and has £84 in fixed costs. Revenue $R = 18q$; cost $C = 84 + 11q$. Profit π means revenue minus cost: $\pi = 18q - (84 + 11q) = 18q - 84 - 11q = 7q - 84$.

Worked example: meet a minimum profit target

Require profit of at least £56: $7q - 84 \geq 56$. Add 84 to both sides: $7q \geq 140$. Divide by positive 7: $q \geq 20$.
Check $q = 20$: revenue = £360, cost = £84 + £220 = £304, profit = £56.
At least 20 whole units are required under the model.



Rearrange formulas before inserting numbers

Rearranging makes a chosen variable the subject, meaning it stands alone on one side. Starting from $C = F + vq$, subtract F : $C - F = vq$. Divide by v : $q = (C - F)/v$, provided $v \neq 0$. To isolate v instead, divide by q : $v = (C - F)/q$, provided $q \neq 0$.

Mathematical answers need domain checks

The domain is the set of values allowed for a variable. Counts of indivisible products are usually non-negative whole numbers; hours or mass may be fractional. An inequality such as $q \geq 6.2$ requires at least 7 whole products. Equations can also have no solution or many solutions, so do not force every problem into one numerical answer.

An algebra routine you can explain

Define each symbol and its units. Translate the words into an equation or inequality. Simplify carefully, apply the same valid operation to both sides, and reverse inequality signs when dividing or multiplying by negatives. Substitute the result into the original relationship. Finally, apply whole-number, non-negativity or other restrictions required by the scenario.

A graph connects two quantities

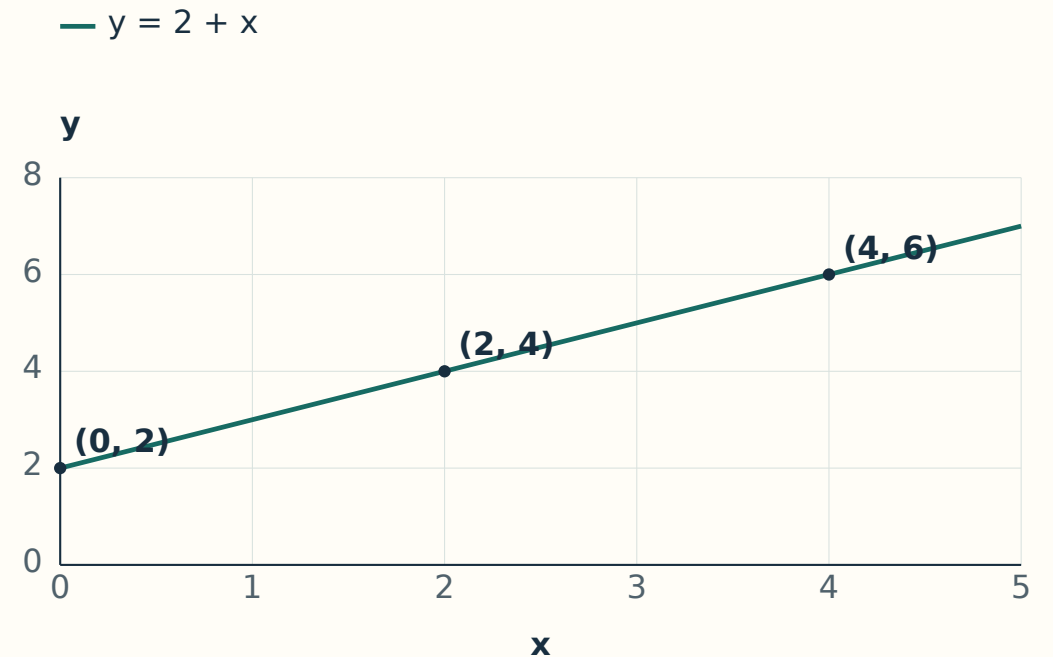
A coordinate graph has a horizontal axis and a vertical axis. Label each axis with its variable and unit. Their crossing is the origin, usually $(0, 0)$. A point represents a pair of values. In this lesson, draw simple graphs on paper as you read; the instructions specify the coordinates and boundaries you need.

Ordered pairs and consistent scales

An ordered pair (x, y) gives the horizontal value first and the vertical value second. To plot $(3, 7)$, move to $x = 3$, then to $y = 7$. Choose equally spaced tick marks on each axis. The two axes may use different scales, but each individual linear scale must stay consistent.

Build points from a formula

For $y = 2 + x$, choose values of x and calculate the matching y values. If $x = 0$, $y = 2$; if $x = 2$, $y = 4$; if $x = 4$, $y = 6$. Plot $(0, 2)$, $(2, 4)$ and $(4, 6)$. A linear relationship places these points on one straight line.

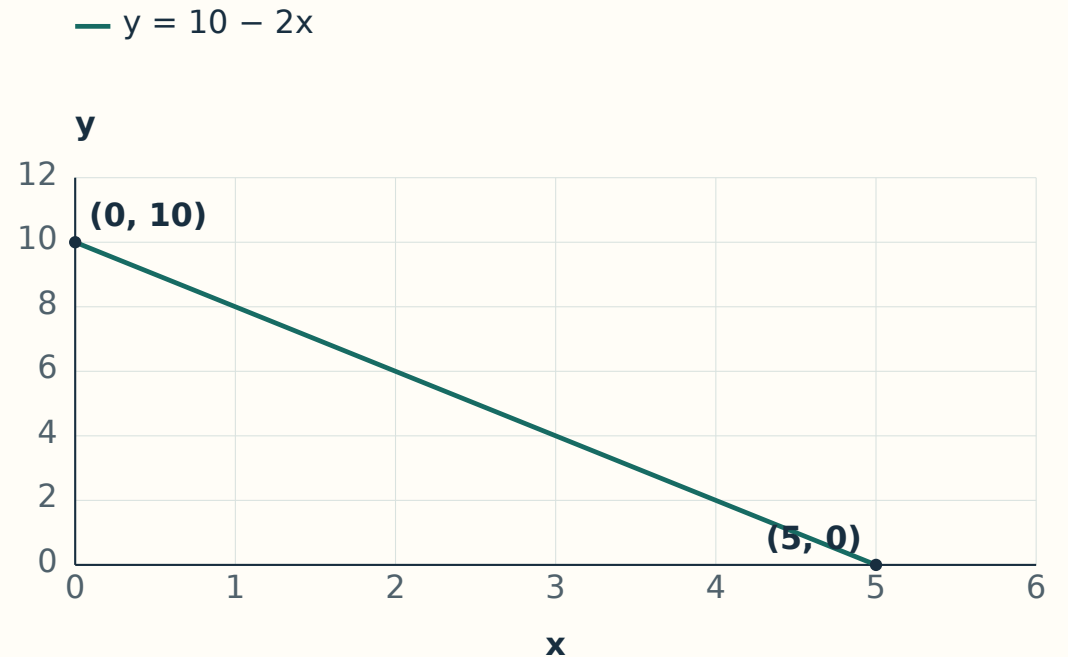


Slope means vertical change per horizontal change

The slope of a non-vertical straight line is $m = (y_2 - y_1)/(x_2 - x_1)$. It measures the change in y for one unit of x . Between $(1, 5)$ and $(4, 11)$, $m = (11 - 5)/(4 - 1) = 6/3 = 2$. Subtract coordinates in the same point order.

Intercepts locate the crossings with axes

An intercept is where a graph crosses an axis. In $y = b + mx$, b is the y-intercept because $x = 0$ gives $y = b$. To find the x-intercept, set $y = 0$ and solve. For $y = 10 - 2x$:
y-intercept = 10; $0 = 10 - 2x$ gives x-intercept = 5.



Positive, negative, zero and undefined slopes

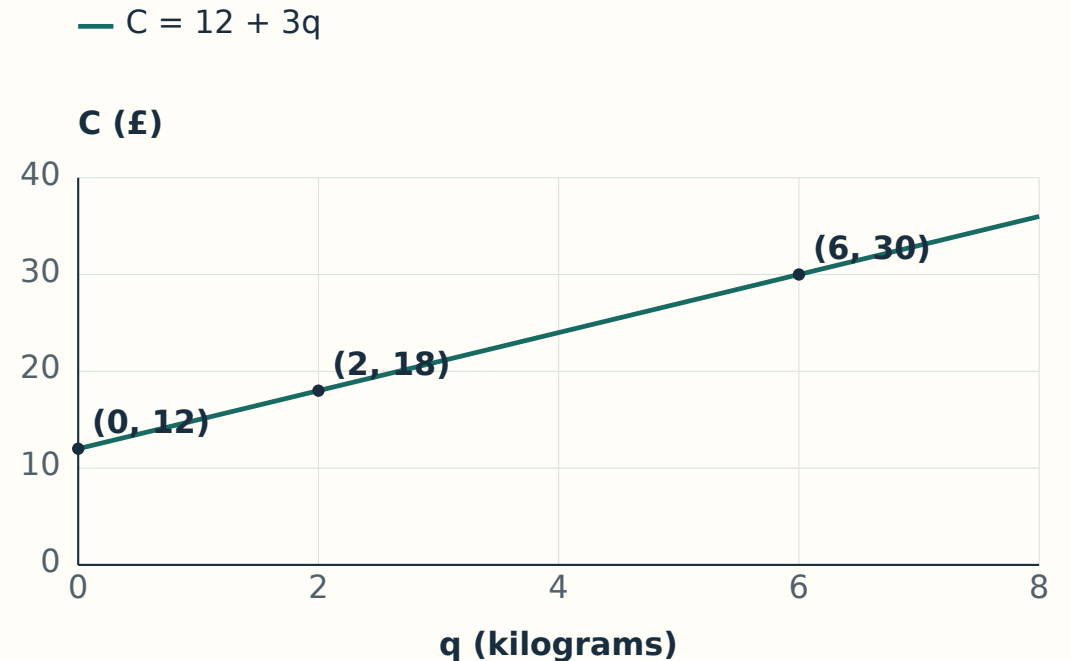
A positive slope rises from left to right; a negative slope falls. A horizontal line has slope zero because y does not change. A vertical line has undefined slope because its horizontal change is zero, which would require division by zero. Visual steepness also depends on axis scales, so compare calculated slopes with their units.

Movement along a line versus a changed line

Changing x within an unchanged equation moves the point along the existing line. Changing an equation's parameter changes the relationship itself. In $y = b + mx$, increasing b shifts the line upwards without changing its slope. Changing m changes the slope. Keep these two kinds of change separate when interpreting economic diagrams.

Worked example: plot an illustrative cost line

A fictional delivery service charges C = 12 + 3 q , where C is pounds and q is kilograms delivered. For $q = 0$: $C = 12 + 3 \times 0 = 12$. For $q = 2$: $C = 12 + 6 = 18$. For $q = 6$: $C = 12 + 18 = 30$. Plot (0, 12), (2, 18), (6, 30).



Worked example: interpret the slope and intercept

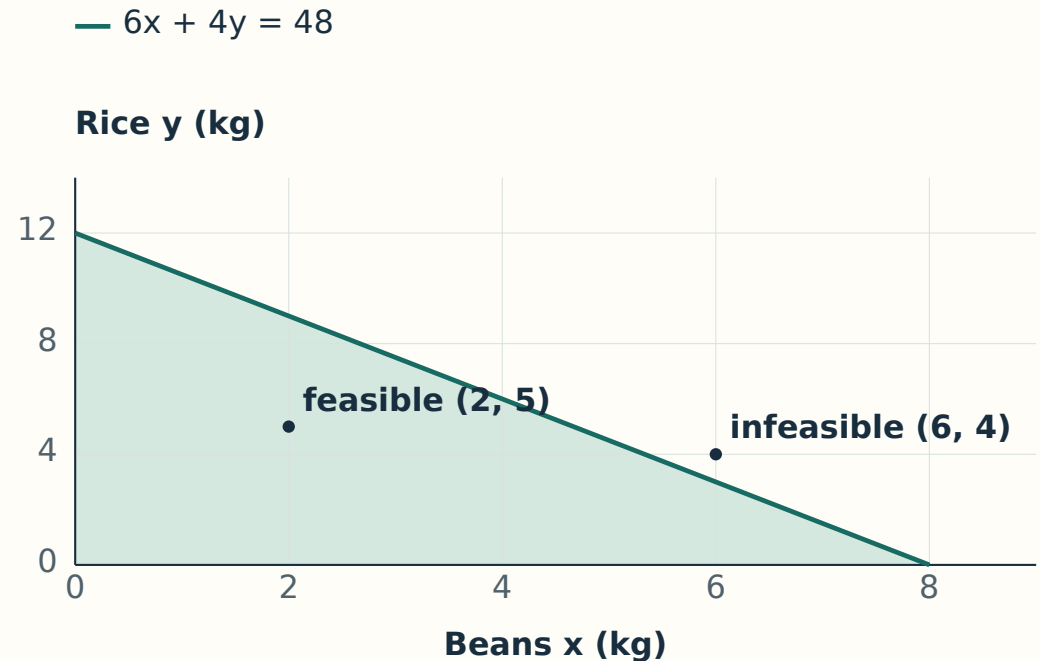
Using (2, 18) and (6, 30), slope = $(30 - 18)/(6 - 2) = 12/4 = \text{£}3$ per kilogram. The vertical intercept is £12. Algebra gives the horizontal intercept: $0 = 12 + 3q$; $-12 = 3q$; $q = -4$. That negative mass lies outside the delivery model's non-negative domain.

Worked example: draw a budget boundary

An illustrative £48 budget buys beans at £6 per kilogram and rice at £4 per kilogram. Let x be kilograms of beans and y kilograms of rice. Affordable combinations satisfy $6x + 4y \leq 48$, with $x \geq 0$ and $y \geq 0$. The full-spending boundary is $6x + 4y = 48$.

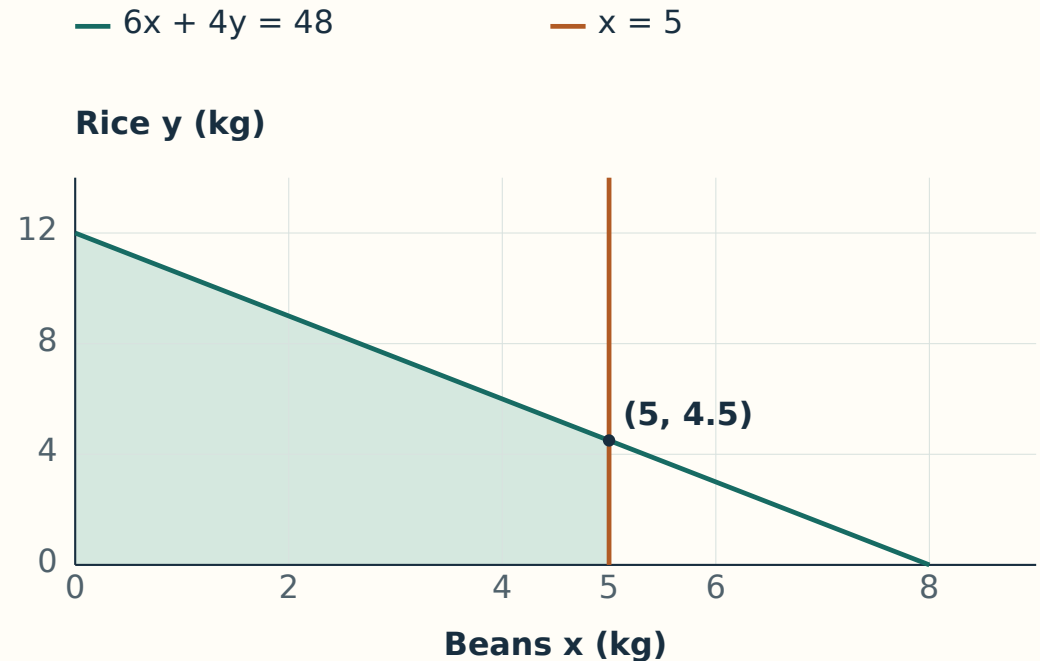
Worked example: choose the affordable side

Boundary intercepts are (8, 0) and (0, 12). Test (2, 5): $\text{cost} = 6 \times 2 + 4 \times 5 = 12 + 20 = \text{£}32 \leq \text{£}48$, so it is feasible. Test (6, 4): $\text{cost} = 36 + 16 = \text{£}52 > \text{£}48$, so it is infeasible. Shade the first-quadrant region on or below the boundary.



Several constraints mean an intersection

A feasible region must satisfy all constraints together. Add $x \leq 5$ to the previous budget model. At $x = 5$, the budget boundary gives $6 \times 5 + 4y = 48$, so $4y = 18$ and $y = 4.5$. The remaining region has corners $(0, 0)$, $(5, 0)$, $(5, 4.5)$, $(0, 12)$.



Read a graph without claiming too much

Check axis labels, units, scale, time period and the range shown. Association means quantities vary together; causation means a change in one produces a change in another. A graph alone usually cannot establish causation. A truncated vertical axis can magnify apparent changes, and extending a line beyond observed or modelled values can be unreliable.

A graph-reading and drawing checklist

Name the variables and units; choose consistent scales; calculate and plot reliable points; identify slope and relevant intercepts; state the domain. For inequalities, draw the boundary and test a point to select the correct side. Combine all restrictions. Finish with a sentence explaining what the picture supports and what it cannot establish.