

Quantities, Units, Fractions and Ratios

Build a reliable language for quantities: attach units, convert measurements, calculate with fractions and divide totals in a stated ratio.

Written walkthrough. Work through each explanation alongside the PDF slides. The lesson page also includes a captioned video with a generic synthetic narrator. It is not a recording of Craig Wright.

Fictional examples teach reasoning rather than describe a real business. No external academic credit is awarded.

Learning objectives

1. Attach appropriate units to quantities and distinguish totals from rates.
2. Convert between related units without changing the underlying amount.
3. Simplify fractions and calculate with a common denominator.
4. Use ratios to allocate a total and use unit rates to compare or scale quantities.

Slide 1: A number needs a meaning

A quantity combines a number with a meaning: £24 is money, 24 kilograms is mass, and 24 hours is time. A unit is the standard used to measure that quantity. In this lesson, every answer should say what is being counted or measured. Our numerical scenarios are invented for learning.

Walkthrough

Begin by separating the calculation from the interpretation. The calculation 12×2 gives 24, but it cannot tell us whether that answer describes pounds, kilograms or hours. We need the question for that. Write the unit beside each starting number, and carry it through to your answer. This small habit prevents many economic mistakes because prices, wages, output and time are different kinds of quantities. You do not need to rush. For each example, first explain in ordinary words what the answer is supposed to describe, then calculate it.

Slide 2: Totals and rates answer different questions

A total describes an amount, such as £48 spent. A rate compares quantities with different units, such as £12 per hour. The word “per” means “for each” and usually signals division. Multiplying a rate by the relevant quantity produces a total: $(£12 \text{ per hour}) \times 4 \text{ hours} = £48$.

Walkthrough

Imagine being told that someone earns twelve pounds. Without a time period, we do not know much about their pay. Twelve pounds per hour gives a rate; forty-eight pounds for four hours gives a total. Notice how the hours in the rate and the hours worked fit together. If we divide the forty-eight-pound total by four hours, we recover twelve pounds per hour. Write rates as a fraction if that helps: pounds divided by hours. Keep the denominator visible, because pounds per hour and hours per pound describe different questions.

Slide 3: Convert units before combining quantities

A conversion changes the numerical description, not the physical amount. One kilogram equals 1,000 grams, so $300 \text{ grams} = 300 \div 1,000 = 0.3 \text{ kilograms}$. One hour equals 60 minutes. Before adding quantities or comparing prices, express the relevant measurements in matching units. Never add kilograms directly to grams.

Walkthrough

Think of conversion as choosing a different-sized measuring unit. A kilogram is a larger unit than a gram, so the same amount needs fewer kilograms than grams. That is why we divide the gram figure by one thousand. Going back from kilograms to grams requires multiplication by one thousand. A quick size check is useful: three hundred grams should be less than one kilogram, so an answer of three hundred thousand kilograms cannot be right. Time needs special care because an hour contains sixty minutes, rather than one hundred. Always write the conversion relationship first.

Slide 4: Fractions describe equal parts

A fraction a/b means a equal parts when the whole has been divided into b equal parts. The numerator a counts the parts; the denominator b sets their size. For $3/4$ of a £20 amount, divide £20 into four equal parts, then take three: $£20 \div 4 \times 3 = £15$.

Walkthrough

The denominator is not just another number to manipulate. It tells you what one part means. If a twenty-pound amount is split into four equal parts, each part is five pounds. Taking three of those parts gives fifteen pounds. The parts must be equal for this interpretation to work. A fraction can also be larger than one: five quarters means five pieces of size one quarter, or one whole and one quarter. When using a fraction in an economic question, identify the whole explicitly. Three quarters of income and three quarters of spending may be different amounts.

Slide 5: Equivalent fractions and decimal notation

Equivalent fractions name the same share. Multiplying or dividing both numerator and denominator by the same non-zero number preserves the value: $6/8 = 3/4$. To write a fraction as a decimal, divide the numerator by the denominator: $3 \div 4 = 0.75$. A terminating decimal ends; some decimal representations continue indefinitely.

Walkthrough

Picture six shaded pieces in a whole cut into eight equal pieces. Pair the small pieces together and you have three shaded pairs out of four pairs. The shaded share has not changed, even though its name has. This is why simplifying requires the same operation on the numerator and denominator. Dividing only the numerator would change the fraction. Decimal notation gives another useful name for the same value. For example, one third is approximately zero point three three three, but rounding it to two decimal places introduces a small difference. Keep extra digits during a calculation and round at the end.

Slide 6: Calculate with fractions carefully

To add fractions, first use a common denominator: $1/2 + 1/3 = 3/6 + 2/6 = 5/6$. To multiply, multiply numerators and denominators: $(2/3) \times (3/5) = 6/15 = 2/5$. To divide by a non-zero fraction, multiply by its reciprocal, which swaps its numerator and denominator.

Walkthrough

Addition combines pieces of the same size, so halves and thirds must first be renamed as sixths. Do not add denominators: doing that would change the size of the pieces while you are counting them. Multiplication answers a different question, such as finding two thirds of three fifths of a whole. Division asks how many of one quantity fit into another. For example, one half divided by one quarter equals one half multiplied by four, which equals two. Two quarters fit inside one half. You cannot divide by zero because there is no meaningful finite answer to that operation.

Slide 7: Ratios compare; rates include units

A ratio compares relative amounts. A red-to-blue ratio of 2:3 means two red items for every three blue items. There are five parts altogether, so red items form $2/5$ of the total. A unit rate gives an amount for one unit, such as pounds per kilogram, and helps compare differently sized purchases.

Walkthrough

Read a ratio in the stated order. Two to three for red to blue does not mean that two thirds of all the items are red. The three counts blue parts, not total parts. Add the two and three to find five parts in the combined whole. A rate also compares quantities, but usually retains different measurement units. If a purchase costs money and has a mass, dividing cost by mass produces pounds per kilogram. That gives a common basis for comparison. Ratios alone show relative size; an actual total or another known quantity is needed to recover absolute amounts.

Slide 8: Worked example: plan a unit-price calculation

An invented flour pack contains 750 grams and costs £1.80. We want its price per kilogram and the cost of 4.5 kilograms at the same rate. First convert the mass: $750 \div 1,000 = 0.75$ kilograms. Then set up the rate: price per kilogram = $£1.80 \div 0.75$ kilograms.

Walkthrough

We start by naming both requested answers. The first must be in pounds per kilogram; the second must be in pounds. The packet mass is given in grams, so it does not yet match the kilogram denominator we want. Divide seven hundred and fifty by one thousand to obtain zero point seven five kilograms. Now divide the packet price by that mass. Do not multiply the price by zero point seven five, because that would not find a price for one kilogram. Pause before calculating and predict that a full kilogram should cost more than this three-quarter-kilogram packet.

Slide 9: Worked example: calculate and verify the cost

Price per kilogram = $1.80 \div 0.75 = 180 \div 75 = \text{£}2.40$ per kilogram. Cost of 4.5 kilograms = $2.40 \times 4.5 = 2.40 \times (4 + 0.5) = 9.60 + 1.20 = \text{£}10.80$. Check another way: $4.5 \div 0.75 = 6$ packs; $6 \times \text{£}1.80 = \text{£}10.80$.

Walkthrough

Multiplying both numbers in the division by one hundred removes the decimal places without changing the quotient. One hundred and eighty divided by seventy-five is two point four, giving a rate of two pounds forty per kilogram. We then multiply that rate by four and a half kilograms. Splitting four and a half into four plus one half makes the arithmetic visible. The independent pack-count check reaches the same total. This example assumes the same price rate applies throughout, with no discounts or extra charges. Six whole packs are available here, so the requested mass is also achievable using this packaging.

Slide 10: Worked example: divide a budget in a ratio

An illustrative £360 budget is divided between rent, food and travel in the ratio 5:3:1. The order matters: rent receives five parts, food three and travel one. Total parts = $5 + 3 + 1 = 9$. Value of one part = $\text{£}360 \div 9 = \text{£}40$.

Walkthrough

A ratio gives instructions for relative shares, rather than fixed pound amounts. Our first task is to discover what one ratio part is worth. Add all the parts because the complete budget must cover all three categories. Nine equal parts share three hundred and sixty pounds, so each part is forty pounds. At this stage, do not assign forty pounds to every category. Rent is entitled to five of these equal parts, while travel gets just one. Writing the category names directly above the ratio numbers is a simple way to preserve the correct ordering throughout the calculation.

Slide 11: Worked example: allocate and check the whole

Rent = $5 \times \text{£}40 = \text{£}200$. Food = $3 \times \text{£}40 = \text{£}120$. Travel = $1 \times \text{£}40 = \text{£}40$. Check the total: $\text{£}200 + \text{£}120 + \text{£}40 = \text{£}360$. Food's fraction of the whole is $3/9 = 1/3$; this agrees with $\text{£}120 \div \text{£}360 = 1/3$.

Walkthrough

We have now translated relative shares into actual money. Two checks are helpful. First, add the three allocations to confirm that the whole budget has been assigned exactly once. Second, compare the allocations with the original ratio. Two hundred to one hundred and twenty to forty simplifies, after dividing each amount by forty, to five to three to one. The food share also illustrates the difference between a ratio comparison and a fraction of the whole. Food to rent is three to five, but food as a share of the complete budget is three out of nine parts.

Slide 12: Direct proportion: scale the same relationship

Two quantities are directly proportional when multiplying one by a factor multiplies the other by the same factor. If four identical tickets cost £18 with no fixed fee, one costs $£18 \div 4 = £4.50$. Seven therefore cost $7 \times £4.50 = £31.50$. The constant unit rate is the key assumption.

Walkthrough

A unit-rate method often feels clearer than a memorised cross-multiplication rule. Reduce the known bundle to one item, then build the new bundle. Four tickets to seven tickets is a multiplication by seven quarters, so multiplying eighteen pounds by seven quarters would give the same result. But the method depends on a constant relationship. A booking fee, bulk discount or minimum charge would change the structure. In economic questions, read the words before assuming proportionality. If there is a fixed fee, total cost might still rise with quantity, but it will not necessarily rise in exactly the same proportion.

Slide 13: Use estimates and sensible precision

An estimate is a deliberately approximate calculation used to check size. Rounding £2.98 to about £3 makes ten items cost about £30. Precision should fit the question: ordinary money totals are usually stated to two decimal places. Keep intermediate values unrounded where possible, and state when an answer is approximate.

Walkthrough

Calculators can produce many digits, but extra digits do not automatically make an answer more meaningful. A purchase total expressed in pounds normally ends at pennies, while a unit price might need extra decimal places for a fair comparison. Estimate before relying on the exact calculation. If ten items cost just under three pounds each, a result near three hundred pounds is a warning. Also distinguish rounding from changing the underlying assumptions. Rounding a price slightly is an approximation; assuming a discount exists is a different model. Explain any assumption that materially affects what your answer means.

Slide 14: Your quantity-checking routine

Use five steps: identify the requested quantity; write the starting units; convert to compatible units; calculate; then check the result against the original situation. Ask whether a fraction refers to the right whole and whether a ratio keeps its stated order. A complete answer combines a value, a unit and a short interpretation.

Walkthrough

You now have a routine that will carry into every later lesson. Before pressing calculator buttons, decide what the answer should measure. During the calculation, keep units and category labels visible. Afterwards, check the total, reverse the operation, or estimate the expected size. These checks are complementary: a tidy arithmetic answer can still use the wrong whole or the wrong unit. In the exercises, show yourself the working on paper even when the submission box asks only for a number. Enter numerical answers without currency symbols where the field already supplies the unit, and use the precision requested by the question.

Questions students ask

Why can I not add $\frac{1}{2}$ and $\frac{1}{3}$ by adding their numerators and denominators?

The denominators describe different-sized pieces. Rename the fractions as $\frac{3}{6}$ and $\frac{2}{6}$ so the pieces match; then count five sixths. Adding denominators would change the piece size without justification.

Does a ratio of 2:3 mean that the first category is $\frac{2}{3}$ of the total?

No. It means the first category is two thirds as large as the second category. The combined total has five equal ratio parts, so the first category is two fifths of that total.

How do I know whether to multiply or divide when converting units?

Write a known relationship first, such as 1 kilogram = 1,000 grams. Moving to the larger unit means fewer units for the same amount, so divide. Check by converting your answer back.

Can I always scale a price using the unit-rate method?

Only when the same rate applies to the quantity in question. Fixed fees, discounts, packaging restrictions and additional charges can change the relationship. State the constant-rate assumption when the scenario uses it.

Your exercises

Submit your answers on the lesson page at craigwright.net/learning. Numerical and choice checks receive server feedback. Written responses are saved ungraded. No assessment answer key appears in this guide.

1. Convert a parcel mass

An invented parcel has a mass of 1,350 grams. Express its mass in kilograms. Enter a decimal number.

Mass (kg)

2. Find a share of a materials fund

A practice project has a £280 materials fund. Exactly $\frac{3}{7}$ of the fund is assigned to timber. How many pounds are assigned to timber?

Timber allocation (£)

3. Allocate three departments a total

An illustrative £420 equipment fund is divided between design, assembly and testing in the ratio 2:4:1, in that order. Enter each allocation in pounds.

Design (£)

Assembly (£)

Testing (£)

4. Compare a different pack size

A fictional 600-gram bag of rice costs £1.74. Find the price per kilogram. Then find the cost of buying 3 kilograms at the same rate, with no additional fees or discounts. Give both money answers to two decimal places.

Price per kilogram (£/kg)

Cost of 3 kilograms (£)

5. Combine compatible shares

Two non-overlapping stages use $\frac{1}{4}$ and $\frac{2}{5}$ of the same material stock. Express the combined share as a fraction in its simplest form. Enter a positive integer numerator and a positive integer denominator.

Numerator

Denominator

Percentages, Changes and Compounding

Calculate shares and changes, distinguish percentage points from percentage changes, and follow changes that build on earlier changes.

Written walkthrough. Work through each explanation alongside the PDF slides. The lesson page also includes a captioned video with a generic synthetic narrator. It is not a recording of Craig Wright.

Fictional examples teach reasoning rather than describe a real business. No external academic credit is awarded.

Learning objectives

1. Convert between percentages, fractions and decimal multipliers.
2. Calculate a percentage change using the correct starting value.
3. Distinguish changes in percentage points from relative percentage changes.
4. Calculate successive and reversed percentage changes, explaining the base used at every step.

Slide 1: Percentage means parts per hundred

A percentage expresses a quantity relative to one hundred equal parts. Thus $25\% = 25/100 = 0.25$. To convert a percentage to a decimal, divide by 100; to convert a decimal to a percentage, multiply by 100. A percentage alone does not identify the size of the underlying whole.

Walkthrough

Percentages are another way to write the fractions from the previous lesson. Twenty-five per cent and one quarter express the same proportion. The hundred is a standard comparison scale, not necessarily the actual number of objects involved. A quarter of twelve items is still twenty-five per cent, even though there are only twelve items. Keep asking, “A percentage of what?” Twenty-five per cent of a small budget and twenty-five per cent of a large budget describe the same share but different amounts of money. In this lesson, all numerical examples are illustrative rather than observations of real prices or returns.

Slide 2: Find a percentage of a quantity

To find $p\%$ of an amount A , calculate $A \times p/100$. For example, 12% of $\pounds 250 = 250 \times 12/100 = 250 \times 0.12 = \pounds 30$. Here $\pounds 250$ is the base: the whole against which the percentage is measured. Label the base before doing the multiplication.

Walkthrough

We can make the arithmetic visible by splitting twelve per cent into ten per cent plus two per cent. Ten per cent of two hundred and fifty pounds is twenty-five pounds. One per cent is two pounds fifty, so two per cent is five pounds. Adding twenty-five and five gives thirty pounds, matching the decimal method. This is a useful mental check, but the decimal method generalises to awkward percentages as well. The percentage and its base work together. Never assume that a percentage reported beside a number must use that number as its base; read the description carefully.

Slide 3: Multipliers include the amount that remains

A multiplier is the factor used to obtain a new value. Increasing A by $p\%$ gives $A \times (1 + p/100)$. Decreasing A by $p\%$ gives $A \times (1 - p/100)$. A 20% increase uses 1.20; a 20% decrease uses 0.80. Multiplying by 0.20 finds only the change amount.

Walkthrough

The number one in each formula represents the entire original amount. With an increase, we keep that original amount and add the extra share. With a decrease, we keep the original amount minus the removed share. This explains why a twenty-per-cent discount leaves eighty per cent of the price rather than twenty per cent. For a positive starting amount, an increase multiplier is above one and a decrease multiplier lies below one. A complete one-hundred-per-cent decrease gives zero. Larger percentage decreases need careful interpretation and are generally unsuitable for ordinary non-negative quantities such as stock remaining.

Slide 4: Percentage change uses the starting value

Percentage change = $[(\text{new value} - \text{original value}) \div \text{original value}] \times 100$. If an illustrative quantity rises from 40 to 50, the change is 10 and the percentage change is $10 \div 40 \times 100 = 25\%$. A negative result indicates a decrease. The usual formula is undefined when the original value is zero.

Walkthrough

Calculate the absolute change first: subtract the original value from the new value. Then compare that change with the original value, not the new one. A rise of ten may be large or small depending on where we started. Ten added to forty is a quarter of the starting amount, hence twenty-five per cent. If the new amount is lower, subtraction gives a negative number and the formula records that direction. For a starting value of zero, division by zero prevents a finite percentage calculation. Report the absolute change instead. Changes involving negative bases also require extra contextual interpretation.

Slide 5: Percentage points compare percentage levels

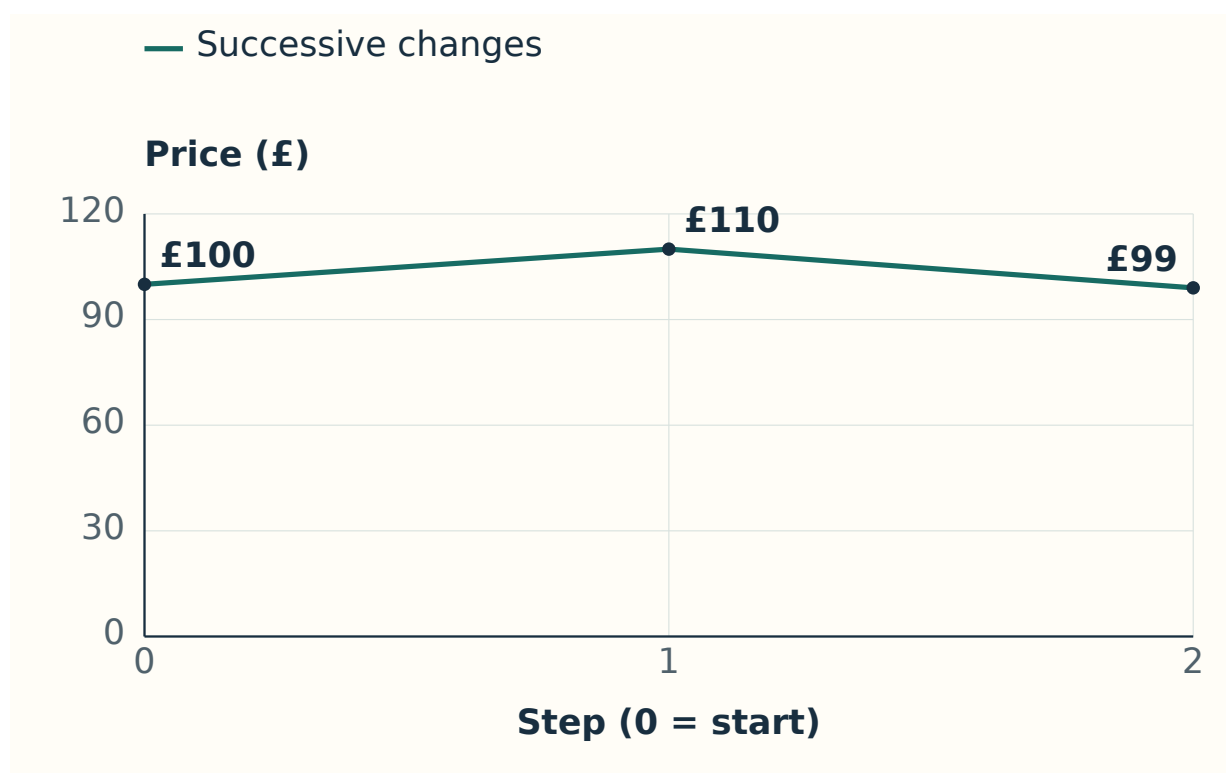
A percentage point is one unit on a percentage scale. If a fictional participation rate rises from 30% to 36%, it rises by $36 - 30 = 6$ percentage points. Relative to its original level, it rises by $(36 - 30) \div 30 \times 100 = 20\%$. These are different descriptions of the same movement.

Walkthrough

The two answers are not competing calculations. They answer different questions. Percentage points give the direct distance between two percentage levels, just as degrees give a distance between temperatures. A percentage change measures that distance relative to the original level. Our increase of six points is one fifth of the original thirty-per-cent level, so its relative size is twenty per cent. Always include the words “percentage points” when you mean a subtraction between percentage levels. Saying only that a rate “rose six per cent” would usually suggest a different relative calculation and could mislead your reader.

Slide 6: Opposite percentages do not usually cancel

An invented price of £100 rises by 10% to £110. A subsequent 10% decrease removes £11, leaving £99. The second percentage uses a different base. The combined multiplier is $1.10 \times 0.90 = 0.99$, so the net movement is a 1% decrease, even though the stated percentages have equal sizes.



The second 10% change starts from £110, so the final value is £99.

Walkthrough

Slow down at the change of base. The first ten per cent is ten per cent of one hundred, which is ten. The second is ten per cent of one hundred and ten, which is eleven. Removing eleven after adding ten leaves us one pound below the start. This is why adding the signed percentages, plus ten and minus ten, gives the wrong answer for successive changes. Multipliers retain the changing base automatically. Equal increases and decreases are not reverse operations. To undo a known multiplier, divide by that multiplier rather than applying the same percentage with the opposite sign.

Slide 7: Compounding builds on earlier changes

Compounding means applying each percentage change to the amount reached after the previous change. At a constant decimal rate r for n periods, final amount = initial amount $\times (1 + r)^n$. The exponent n means repeated multiplication. With varying rates, multiply the separate period multipliers in sequence.

Walkthrough

An exponent is a compact way to record repeated multiplication: a multiplier squared means the multiplier times itself. Three periods require three copies. The percentage changes may describe output, costs or another quantity; compounding is a mathematical structure, not a promise about real-world performance. It differs from repeatedly adding the same original-base amount. Before using the formula, identify the length of each period and make sure the rate refers to that same period. An annual percentage cannot simply be relabelled as a monthly percentage. Fees, withdrawals, capacity limits or changing rates would need to be included separately in the model.

Slide 8: Worked example: discount an illustrative price

A fictional item lists at £120. A 15% discount is applied first, followed by a 5% charge on the discounted price. Discount amount = $£120 \times 15/100 = £18$. Discounted price = $£120 - £18 = £102$. Equivalently, $£120 \times 0.85 = £102$. The next calculation must use £102 as its base.

Walkthrough

This question specifies the order and the base of the charge, so we do not need to guess how the seller calculates it. Begin with the discount. Fifteen divided by one hundred is zero point one five, and multiplying one hundred and twenty by that decimal gives eighteen pounds. Subtract eighteen from one hundred and twenty to get one hundred and two pounds. The multiplier route reaches the same intermediate value because eighty-five per cent remains. Write that intermediate value down clearly. The five-per-cent charge is attached to this discounted amount, rather than to the original list price.

Slide 9: Worked example: apply the charge and compare

Charge = $£102 \times 0.05 = £5.10$. Final price = $£102 + £5.10 = £107.10$. Combined multiplier = $0.85 \times 1.05 = 0.8925$. Net change = $(107.10 - 120) \div 120 \times 100 = -12.90 \div 120 \times 100 = -10.75\%$. The final price is 10.75% below the original price.

Walkthrough

Five per cent of one hundred and two pounds is five pounds ten, so adding the charge produces one hundred and seven pounds ten. We then compare that final total with the original list price. The reduction is twelve pounds ninety, which is ten point seven five per cent of one hundred and twenty pounds. The combined multiplier provides a second check: zero point eight nine two five means the final amount is eighty-nine point two five per cent of the original. Its distance below one hundred per cent is ten point seven five percentage points on that original-price index.

Slide 10: Worked example: set up repeated growth

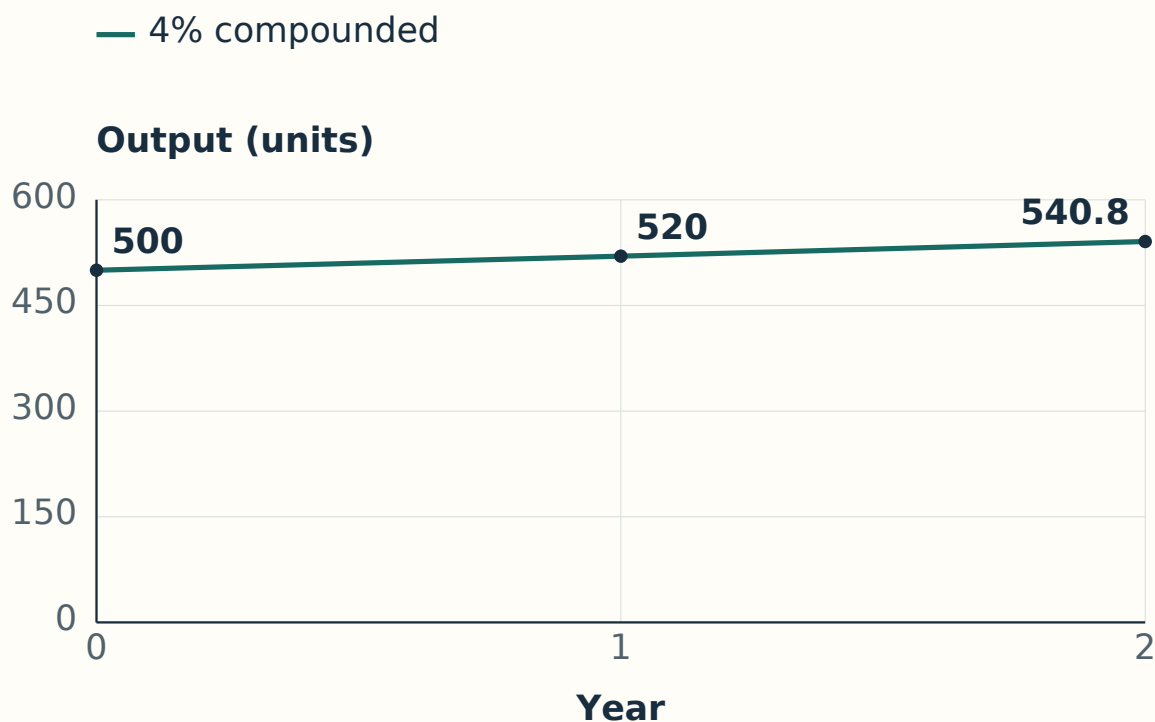
An invented planning model starts with 500 units of output. Output grows by 4% in each of two years, with each increase based on the previous year's output. The annual multiplier is $1 + 4/100 = 1.04$. After year one: $500 \times 1.04 = 520$ units. The second year starts from 520.

Walkthrough

This is a deliberately simple model with a constant growth rate and no rounding to whole units. The units could describe a divisible measure of output, so fractional values are allowed. First turn four per cent into the decimal zero point zero four, then add one to include the original output. Multiplying five hundred by one point zero four gives five hundred and twenty. You can check that by finding four per cent of five hundred, which is twenty, and adding it to the start. The crucial next step is to use five hundred and twenty as the new base.

Slide 11: Worked example: finish and measure total growth

Year-two increase = $520 \times 0.04 = 20.8$ units. Final output = $520 + 20.8 = 540.8$ units. Equivalently, $500 \times 1.04^2 = 500 \times 1.0816 = 540.8$. Overall percentage growth = $(540.8 - 500) \div 500 \times 100 = 8.16\%$. Adding 4% + 4% would miss growth on the earlier increase.



Output grows by 20 units first, then 20.8 units because the base has grown.

Walkthrough

The second increase is twenty point eight units, slightly larger than the first increase of twenty. That additional zero point eight is four per cent of the first year's extra twenty units. Adding the second increase to five hundred and twenty gives five hundred and forty point eight. To find the overall rate, return to the original five hundred-unit base. The gain of forty point eight divided by five hundred is zero point zero eight one six, or eight point one six per cent. Adding two lots of twenty would produce five hundred and forty, which corresponds to a different, non-compounded rule.

Slide 12: Reverse a percentage change by dividing

If a value after a 15% increase is £69, the original value satisfies $\text{original} \times 1.15 = 69$. Therefore $\text{original} = 69 \div 1.15 = £60$. Subtracting 15% of £69 would use the wrong base. For a known discount, divide the final value by the fraction of the original that remains.

Walkthrough

Working backwards means undoing the operation that produced the observed value. A fifteen-per-cent increase multiplies the original by one point one five, so division by one point one five reverses it. Sixty pounds is the result, and multiplying sixty by one point one five checks the answer. Do not remove fifteen per cent of sixty-nine: that would reverse a different operation based on sixty-nine itself. For a discount, the same logic applies with a multiplier below one. If a full one-hundred-per-cent discount leaves zero, however, the original cannot be recovered from that final zero alone.

Slide 13: Report the base, period and rounding

A useful percentage statement identifies its base, its time period and whether the change is relative or measured in percentage points. Round only the final answer unless instructed otherwise. A large percentage increase can describe a small absolute change, so report the underlying amounts when they matter. Percentages alone cannot establish why a change occurred.

Walkthrough

Suppose a tiny activity doubles while a much larger activity grows only slightly. The tiny activity has the larger percentage increase, but it may still add fewer units. This is why absolute changes and relative changes often belong together. Time periods matter too: a growth figure for one month cannot be compared fairly with a whole-year figure without additional analysis. When rounding a repeated calculation, keep the full intermediate values so that early rounding does not accumulate. Finally, a percentage describes a relationship between numbers. It does not show whether a policy, a price change or some other event caused the movement.

Slide 14: A reliable percentage decision tree

Ask what the question wants. A share requires $\text{base} \times \text{decimal percentage}$. A new amount requires a multiplier. A percentage change requires $\text{change} \div \text{original} \times 100$. A change between percentage levels may require percentage points. Successive changes require multiplied factors. Recovering an original value requires dividing by the known multiplier.

Walkthrough

Before attempting the exercises, practise choosing the operation in words. Are you finding part of a whole, moving a quantity forwards, comparing endpoints, or working backwards? Those are different tasks even when the same percentage appears in the question. Write the base next to each calculation and label percentage-point answers explicitly. For successive changes, a small timeline can help: starting amount, first new amount, final amount. Check the final answer against a combined multiplier. Enter a percentage such as seven point five as 7.5 when the field already displays the percent unit, rather than entering its decimal multiplier.

Questions students ask

Why do we divide percentage change by the original value?

The question measures the movement relative to the starting position. Dividing by the final value answers a different question. Label the starting value before calculating, especially when a change is reversed.

Are percentage points just a more formal name for percentages?

No. Percentage points measure the direct difference between percentage levels. A percentage change divides that difference by the original level and expresses the result per hundred.

Can a percentage exceed 100%?

Yes. A quantity can be more than the whole used for comparison, and an increase can exceed the original amount. But a share of one ordinary non-negative whole cannot exceed 100% without overlapping categories or a different interpretation.

Does changing the order of two percentage multipliers always change the result?

Two pure multiplicative changes on the evolving amount give the same final product in either order. However, fixed charges, thresholds, rounding rules or percentages attached to specified original bases can make real arrangements behave differently.

Why keep extra digits until the end?

Rounding changes the value slightly. Reusing a rounded intermediate value can carry that difference into later steps. Keep the available precision during the calculation, then round to the accuracy the question requests.

Your exercises

Submit your answers on the lesson page at craigwright.net/learning. Numerical and choice checks receive server feedback. Written responses are saved ungraded. No assessment answer key appears in this guide.

1. Calculate a maintenance share

An invented £350 budget allocates 18% to maintenance. How many pounds are allocated to maintenance?

Maintenance allocation (£)

2. Measure a price increase

A fictional price rises from £84 to £96. Give the absolute increase in pounds, then the percentage increase relative to the original price. Round the percentage to two decimal places.

Absolute increase (£)

Percentage increase (%)

3. Distinguish two descriptions of a rate change

In an invented survey, a participation rate rises from 12% to 15%. Give the increase in percentage points and the relative percentage increase. Both requested increases are positive.

Increase (percentage points)

Relative increase (%)

4. Apply successive changes

An illustrative price begins at £240. It increases by 10%, then decreases by 5% of the increased price. Find the final price and the net percentage change from the starting price. Enter a positive net percentage for an increase or a negative one for a decrease. Give the price to two decimal places.

Final price (£)

Net percentage change (%)

5. Recover a price before a discount

A fictional item has a price of £64 after a 20% discount. What was the price immediately before that discount? There are no other charges.

Original price (£)

Variables, Equations and Economic Formulas

Translate short economic descriptions into algebra, solve linear equations and inequalities, rearrange formulas and check whether a solution makes sense.

Written walkthrough. Work through each explanation alongside the PDF slides. The lesson page also includes a captioned video with a generic synthetic narrator. It is not a recording of Craig Wright.

Fictional examples teach reasoning rather than describe a real business. No external academic credit is awarded.

Learning objectives

1. Distinguish variables, constants, expressions and equations.
2. Substitute numerical values and use brackets and order of operations correctly.
3. Solve linear equations and inequalities while preserving valid solutions.
4. Rearrange simple cost and profit formulas, respecting units and domain restrictions.

Slide 1: Letters stand for quantities

A variable is a symbol whose value can change or is not yet known. A constant is held fixed within the problem. In $C = 18 + 6q$, q represents quantity and C represents total cost in pounds. The fixed charge is £18 and the cost per unit is £6. Writing $6q$ means $6 \times q$.

Walkthrough

Algebra does not replace arithmetic with a new kind of mystery. It lets us describe the same calculation before choosing particular numbers. If quantity changes, the formula tells us how total cost changes. A letter needs a definition just as a number needs a unit. Here q counts units and C measures pounds, so the coefficient six has the meaning pounds per unit. A coefficient is the number multiplying a variable. Constants are fixed only within the stated model: a different supplier or a different period could have a different fixed charge or unit cost.

Slide 2: Expressions, equations and solutions

An expression, such as $3x + 7$, describes a calculation. An equation, such as $3x + 7 = 22$, states that two expressions have equal values. A solution is a value that makes the equation true. A linear equation contains variables only to the first power, with no products between variables.

Walkthrough

An expression has no equals sign and does not by itself ask you to find a particular value. An equation adds a condition. Our example asks which value of x makes three times x plus seven equal twenty-two. The answer is not established merely by producing a number; it must satisfy the condition when substituted back. “Linear” will later connect to a straight-line graph. For now, notice that x squared and a product such as x times y would take us beyond the simple linear relationships in this lesson. We begin with one unknown so each step is easy to inspect.

Slide 3: Brackets and the order of operations

Calculate brackets first, then powers, then multiplication and division, then addition and subtraction. Operations at the same level are evaluated from left to right. Distributing multiplication gives $3(x + 2) = 3x + 6$. Collect like terms only: $4x + 2x = 6x$, but $4x + 2$ cannot be combined into $6x$.

Walkthrough

Brackets show which pieces belong together. Three times the whole expression x plus two means three copies of x and three copies of two. Missing the second multiplication is a common error. Like terms contain the same variable part, so four x plus two x counts six copies of x . A plain two is a different kind of term and stays separate. To see why, try x equal to ten: four x plus two gives forty-two, whereas six x gives sixty. For division involving a whole expression, use brackets explicitly, as in $(C \text{ minus } F) \text{ divided by } v$.

Slide 4: Substitution turns a formula into arithmetic

Substitution means replacing a symbol with its stated value. For $C = 18 + 6q$ and $q = 4$, calculate $C = 18 + 6 \times 4 = 18 + 24 = £42$. Multiply before adding. Check units: pounds plus pounds per unit multiplied by units gives pounds, as required for total cost.

Walkthrough

Write a separate substitution line rather than trying to jump directly to the answer. This makes the chosen value and the order of operations visible. Six times four comes before adding eighteen because multiplication takes priority over addition. The unit check is equally important: six pounds per unit times four units gives twenty-four pounds, which can legitimately be added to the eighteen-pound fixed charge. If your formula tried to add pounds directly to a count of units, that would signal a modelling problem even if the arithmetic were possible. Substitution is also the main tool for checking an equation after solving it.

Slide 5: Preserve the balance of an equation

To solve an equation, use operations that preserve equality. Add or subtract the same quantity on both sides; multiply or divide both sides by the same non-zero number. For $3x + 7 = 22$, subtract 7: $3x = 15$. Divide by 3: $x = 5$. Check: $3 \times 5 + 7 = 22$.

Walkthrough

Imagine the equals sign as a balanced relationship. Changing just one side generally destroys that balance, while applying the same valid operation to both sides preserves it. We undo addition before multiplication here because seven was added after three multiplied x . Subtracting seven removes that outer operation, leaving three x . Dividing by three isolates x . “Move it across and change the sign” can be a shorthand, but it hides the reason the step works. Write the operation on both sides while learning. Dividing by zero is never valid, and dividing by an unknown expression requires checking whether it could be zero.

Slide 6: Unknowns can appear on both sides

For $5x + 7 = 2x + 22$, subtract $2x$ from both sides: $3x + 7 = 22$. Subtract 7: $3x = 15$. Divide by 3: $x = 5$. Check both original sides: $5 \times 5 + 7 = 32$ and $2 \times 5 + 22 = 32$.

Walkthrough

The same balance rule works when x appears on both sides. Subtracting two x removes the right-hand variable term and leaves three x on the left. We then proceed as before. The checking stage should use the original equation, because an accidental mistake in an intermediate line could otherwise be carried into the check. Both original sides evaluate to thirty-two, so the condition holds. You could instead subtract five x first and work with negative coefficients. That route is also valid. Choose a clear route, but remember that correctness comes from preserving equality rather than following one compulsory sequence.

Slide 7: Inequalities describe ranges of answers

An inequality compares sizes: $<$ means less than, $\leq$ means less than or equal to, $>$ means greater than, and $\geq$ means greater than or equal to. Adding the same quantity preserves the sign. Multiplying or dividing by a negative number reverses it: $-2x < 6$ becomes $x > -3$ after division by -2 .

Walkthrough

An equation often identifies a particular value, while an inequality can identify a whole range. The strict symbols exclude the boundary; the symbols with the extra line include it. The sign reversal is easiest to understand on the number line. Two is less than five, but multiplying both by minus one gives minus two, which is greater than minus five. Negative multiplication reverses order. In our algebra example, division by minus two therefore turns “less than” into “greater than”. Test a value from your proposed range in the original inequality. Zero works here, while minus four does not, which supports the direction we found.

Slide 8: Worked example: turn a cost question into an equation

A fictional workshop charges £18 for setup and £6 per unit. A total bill is £90. Let q be the number of units, so $18 + 6q = 90$. Subtract 18 from both sides: $6q = 90 - 18 = 72$. The £72 remaining covers only the per-unit charges.

Walkthrough

We already know the structure of this cost model, but now the total is known and quantity is unknown. State q clearly before forming the equation. The setup charge appears once, while six pounds is charged for each unit. Subtracting eighteen pounds removes the fixed component from both sides. This leaves seventy-two pounds available to cover the quantity-dependent component. Do not divide ninety by six immediately, because that would incorrectly treat the setup charge as if it purchased units. Keep the economic interpretation beside the algebra: six pounds per unit multiplied by q units must equal the remaining seventy-two pounds.

Slide 9: Worked example: solve, check and interpret

From $6q = 72$, divide both sides by 6: $q = 72 \div 6 = 12$ units. Check the original bill: $£18 + £6 \times 12 = £18 + £72 = £90$. The result is a whole, non-negative quantity, so it fits this workshop model as well as satisfying the equation.

Walkthrough

Dividing the remaining seventy-two pounds by six pounds per unit yields twelve units. The pounds cancel in the rate calculation, leaving a count of units rather than another money total. Now rebuild the original bill: twelve units cost seventy-two pounds, and adding the eighteen-pound setup charge gives ninety pounds. This independent reconstruction is stronger than simply repeating the division. We also check the permitted domain, meaning the values the variable is allowed to take. If the workshop sells only complete units, a negative answer or a fraction of a unit would need further interpretation rather than automatic acceptance.

Slide 10: Worked example: model revenue, cost and profit

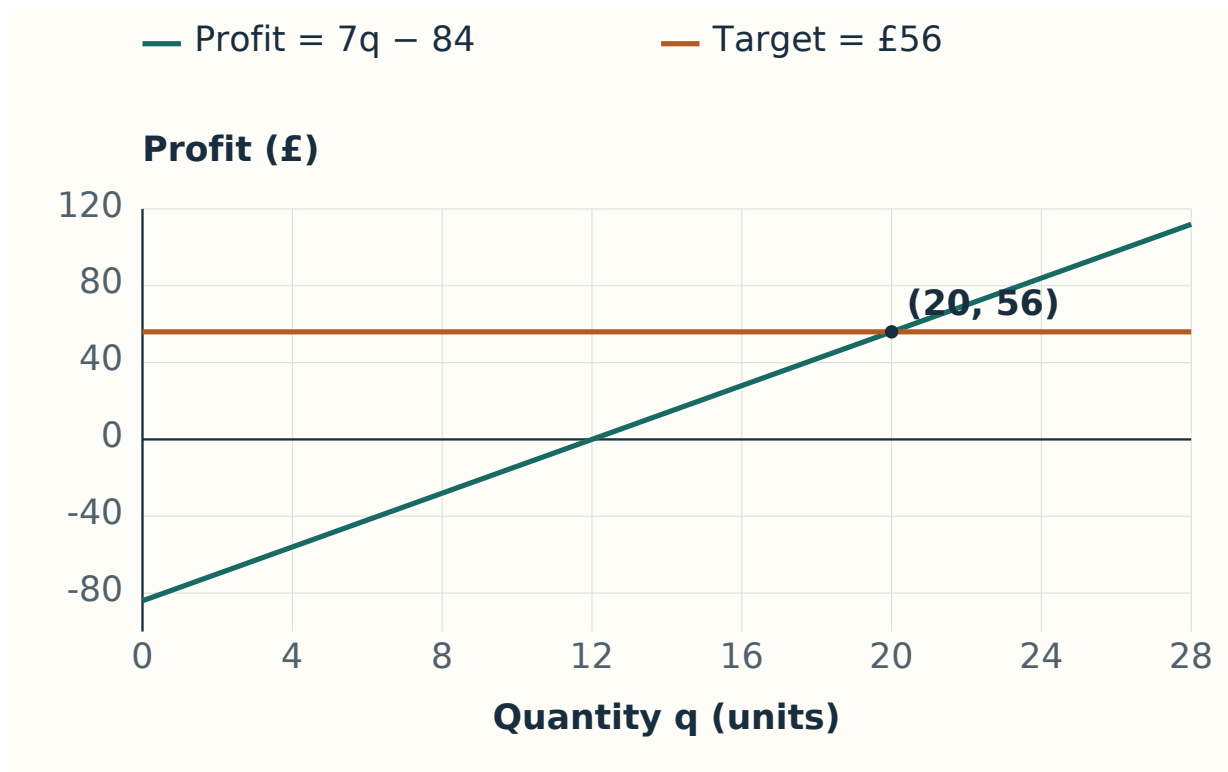
An invented business receives £18 per unit sold, pays £11 per unit in variable costs and has £84 in fixed costs. Revenue $R = 18q$; cost $C = 84 + 11q$. Profit π means revenue minus cost: $\pi = 18q - (84 + 11q) = 18q - 84 - 11q = 7q - 84$.

Walkthrough

The Greek letter pi is simply a label for profit here; it is not the circle constant in this formula. Revenue is money received from sales. Variable cost changes with output, while fixed cost is held constant over the modelled range. Subtract the whole cost expression from revenue, which is why the brackets matter. Removing the brackets changes both cost terms to negatives. Collecting eighteen q minus eleven q leaves seven q , then subtract the fixed eighty-four pounds. Each additional unit contributes seven pounds towards fixed cost and then profit, assuming every unit produced is sold at the stated price.

Slide 11: Worked example: meet a minimum profit target

Require profit of at least £56: $7q - 84 \geq 56$. Add 84 to both sides: $7q \geq 140$. Divide by positive 7: $q \geq 20$. Check $q = 20$: revenue = £360, cost = £84 + £220 = £304, profit = £56. At least 20 whole units are required under the model.



Profit reaches the £56 target at 20 units; larger feasible quantities exceed it.

Walkthrough

The phrase “at least” includes the target itself, so we use greater than or equal to rather than a strict inequality. Adding eighty-four to both sides gives seven q greater than or equal to one hundred and forty. We divide by positive seven, so the direction stays unchanged. Twenty is the smallest permitted whole quantity. Reconstructing revenue and cost at twenty verifies the boundary. At nineteen units, the formula gives forty-nine pounds of profit, below the target, so nineteen is insufficient. The conclusion depends on the model assumptions, including constant prices, constant unit costs and no unsold output or capacity restriction.

Slide 12: Rearrange formulas before inserting numbers

Rearranging makes a chosen variable the subject, meaning it stands alone on one side. Starting from $C = F + vq$, subtract F : $C - F = vq$. Divide by v : $q = (C - F)/v$, provided $v \neq 0$. To isolate v instead, divide by q : $v = (C - F)/q$, provided $q \neq 0$.

Walkthrough

The letters now represent general quantities: C is total cost, F is fixed cost, v is variable cost per unit and q is quantity. The same balance operations apply even when the numbers are not yet supplied. Brackets around C minus F are essential: the whole difference is divided by v or q. Making a general formula first can reduce repeated work when several scenarios use the same structure. State the non-zero condition for each denominator. If no units are produced, total cost alone cannot reveal variable cost per unit, because the variable-cost term contributes nothing regardless of its coefficient.

Slide 13: Mathematical answers need domain checks

The domain is the set of values allowed for a variable. Counts of indivisible products are usually non-negative whole numbers; hours or mass may be fractional. An inequality such as $q \geq 6.2$ requires at least 7 whole products. Equations can also have no solution or many solutions, so do not force every problem into one numerical answer.

Walkthrough

Suppose an algebraic lower bound is six point two and products cannot be divided. Six is too small, so round upwards to seven, rather than using ordinary nearest-integer rounding. An upper bound of six point two would allow at most six whole products. Context decides the direction. Sometimes simplification removes the unknown entirely. If it leaves zero equals three, no value can satisfy the original equation. If it leaves zero equals zero, every value in the permitted domain satisfies it, provided no earlier operation excluded a valid case. Checking these outcomes prevents a calculator-looking answer from replacing actual reasoning.

Slide 14: An algebra routine you can explain

Define each symbol and its units. Translate the words into an equation or inequality. Simplify carefully, apply the same valid operation to both sides, and reverse inequality signs when dividing or multiplying by negatives. Substitute the result into the original relationship. Finally, apply whole-number, non-negativity or other restrictions required by the scenario.

Walkthrough

The aim is not to memorise a separate trick for each question. A small set of balance rules solves many problems once the situation is represented accurately. Read words such as “exactly”, “at most” and “at least” carefully: they determine whether you need equality or an inequality, and which direction it faces. Keep one operation per line until the process feels natural. For the exercises, retain your working even when the form asks only for a boundary or a number. You should be able to say why the result works and why values outside an inequality range fail the condition.

Questions students ask

Is $6q$ the same as $6 + q$?

No. Writing a coefficient beside a variable means multiplication. If q is four, $6q$ is twenty-four, while $6 + q$ is ten. Write the multiplication sign explicitly whenever the shorthand feels unclear.

Why does subtracting a bracket change every term inside it?

Subtracting an expression is multiplying that entire expression by negative one. Thus $a - (b + c) = a - b - c$. Apply the negative to both terms, rather than only the first.

Must I reverse an inequality whenever I see a negative number?

No. Reverse the sign only when multiplying or dividing both sides by a negative quantity. Adding or subtracting a negative number does not by itself reverse the ordering.

Why might a correct algebraic result still be unsuitable?

The situation can impose additional restrictions. Products may need whole-number counts, quantities may be non-negative, and a cost formula may apply only within a stated capacity. Check those conditions after solving.

Can I divide both sides by an unknown variable?

Only after accounting for the possibility that it is zero. Dividing by a variable without checking can discard a valid zero solution or perform an undefined operation. In these introductory formulas, state a non-zero assumption whenever it is needed.

Your exercises

Submit your answers on the lesson page at craigwright.net/learning. Numerical and choice checks receive server feedback. Written responses are saved ungraded. No assessment answer key appears in this guide.

1. Solve an equation with two variable terms

Solve $4x + 11 = 2x + 35$. Enter the value of x that makes the original equation true.

Value of x

2. Find the largest affordable quantity

A fictional order costs £25 in fixed charges plus £8 per unit. The total must be no more than £129. Units are indivisible and quantities cannot be negative. What is the largest affordable whole-number quantity?

Maximum quantity (units)

3. Reach a profit target in a new scenario

An illustrative seller receives £16 per unit, incurs £10 in variable cost per unit and has fixed costs of £90. Every unit produced is sold. What is the minimum whole-number quantity needed for profit of at least £78?

Minimum quantity (units)

4. Recover the variable cost per unit

Use $C = F + vq$. In a fictional production run, $C = £174$, $F = £30$ and $q = 12$ units. Rearrange the relationship to find v , then enter the variable cost per unit.

Variable cost per unit (£/unit)

5. Handle a negative coefficient

Solve $-3x + 8 \geq 20$. Report the solution in the form x [relation] boundary. Select the correct relation and enter the boundary value; the relation is read with x on the left.

Relation

1. $<$

2. $\leq$

3. $>$

4. $\geq$

Boundary value

Coordinates, Straight Lines and Feasible Regions

Connect algebra to a graph, calculate slopes and intercepts, identify combinations that satisfy constraints and separate a graphical pattern from a causal explanation.

Written walkthrough. Work through each explanation alongside the PDF slides. The lesson page also includes a captioned video with a generic synthetic narrator. It is not a recording of Craig Wright.

Fictional examples teach reasoning rather than describe a real business. No external academic credit is awarded.

Learning objectives

1. Read and plot ordered pairs using labelled axes and consistent scales.
2. Calculate and interpret the slope and intercepts of a straight line.
3. Represent linear inequalities as feasible regions with included or excluded boundaries.
4. Distinguish movements along a line, shifts of a relationship and unsupported causal interpretations.

Slide 1: A graph connects two quantities

A coordinate graph has a horizontal axis and a vertical axis. Label each axis with its variable and unit. Their crossing is the origin, usually $(0, 0)$. A point represents a pair of values. In this lesson, draw simple graphs on paper as you read; the instructions specify the coordinates and boundaries you need.

Walkthrough

A graph is another language for the relationships we have been calculating. One position records two values at once, allowing us to see patterns that are harder to notice in a list. Start every drawing by naming the quantities on the axes. “Quantity, in kilograms” and “cost, in pounds” are more useful labels than x and y alone. We conventionally put x horizontally and y vertically, but economic graphs may use other variable names or particular axis conventions. Read the labels rather than assuming. Every numerical graph in this lesson is an invented teaching model, not evidence about an actual market.

Slide 2: Ordered pairs and consistent scales

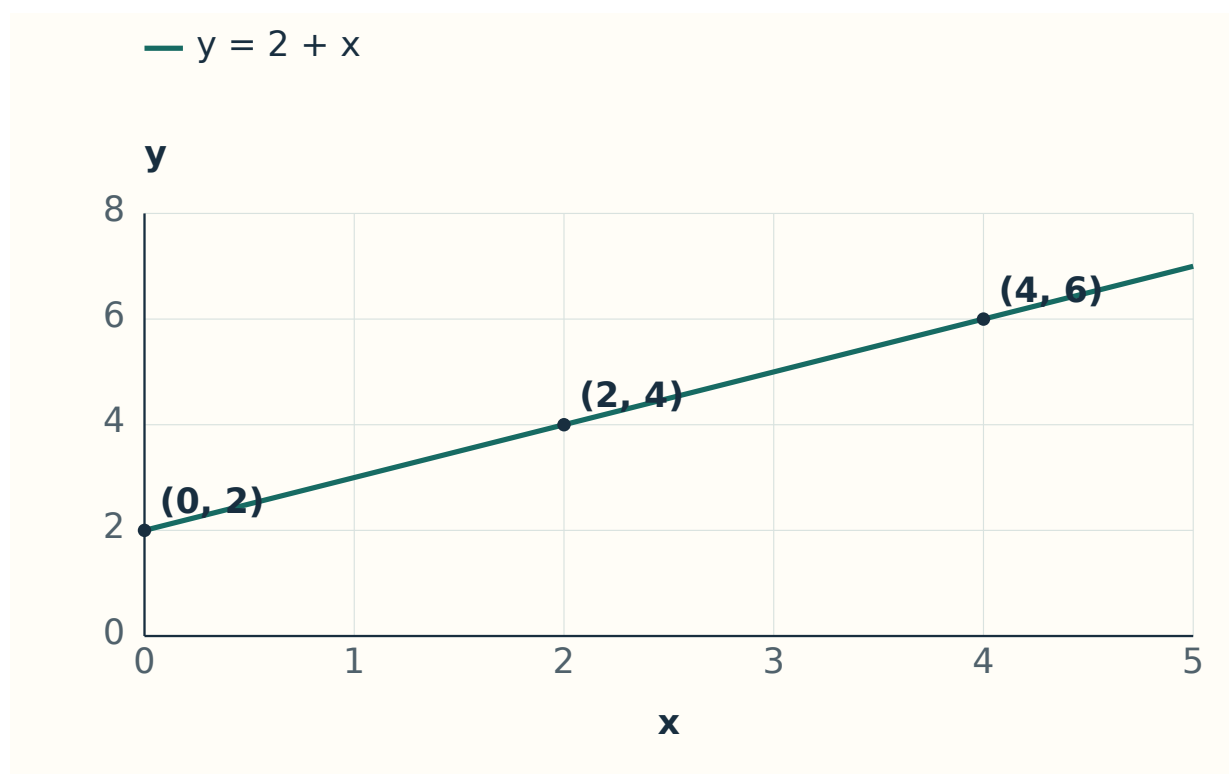
An ordered pair (x, y) gives the horizontal value first and the vertical value second. To plot $(3, 7)$, move to $x = 3$, then to $y = 7$. Choose equally spaced tick marks on each axis. The two axes may use different scales, but each individual linear scale must stay consistent.

Walkthrough

“Ordered” matters because reversing the entries usually places a point somewhere else. Three across and seven up is different from seven across and three up. If negative values are allowed, move left for negative x and down for negative y . Choose scales that fit the relevant values comfortably on the page. For example, each horizontal tick could represent one unit while each vertical tick represents five pounds. That is perfectly valid when labelled clearly. However, switching halfway along one axis from increments of five to increments of ten would distort an ordinary linear graph unless a special break or different scale were explicitly explained.

Slide 3: Build points from a formula

For $y = 2 + x$, choose values of x and calculate the matching y values. If $x = 0$, $y = 2$; if $x = 2$, $y = 4$; if $x = 4$, $y = 6$. Plot $(0, 2)$, $(2, 4)$ and $(4, 6)$. A linear relationship places these points on one straight line.



Plot the calculated pairs; the line $y = 2 + x$ passes through all three.

Walkthrough

A small value list is a bridge between algebra and a graph. Substitute each horizontal value into the formula and preserve the matching pair. Three points are useful because the third can reveal an arithmetic or plotting error, even though two distinct points determine a straight line. If the variable can take every intermediate value, connect the points with a straight line over the permitted range. If only whole-number quantities are possible, individual admissible points have the literal meaning; the connecting line is a useful model guide. Do not extend a line beyond the domain without checking whether the formula still applies there.

Slide 4: Slope means vertical change per horizontal change

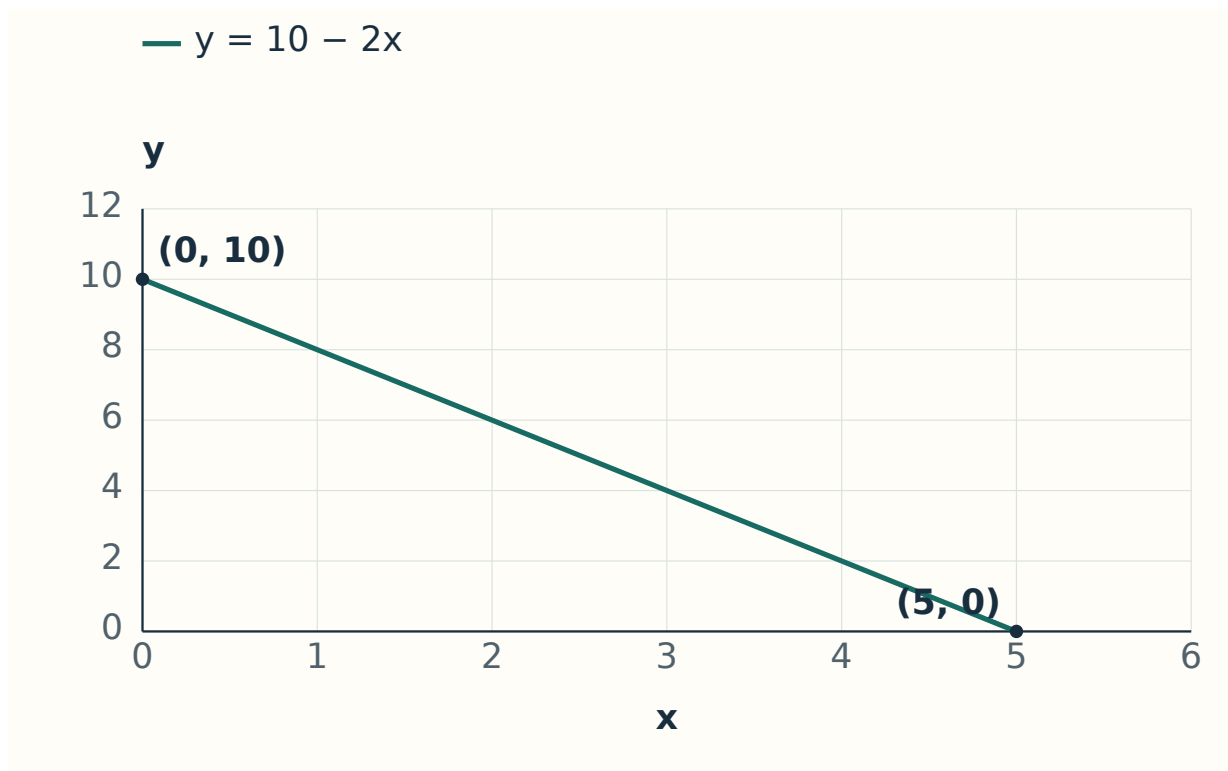
The slope of a non-vertical straight line is $m = (y_2 - y_1)/(x_2 - x_1)$. It measures the change in y for one unit of x . Between $(1, 5)$ and $(4, 11)$, $m = (11 - 5)/(4 - 1) = 6/3 = 2$. Subtract coordinates in the same point order.

Walkthrough

The subscripts simply label the first and second points. They do not mean multiplication or powers. We calculate the vertical difference and divide it by the horizontal difference. In this example, moving three units right raises the vertical value by six, so each horizontal unit corresponds to two vertical units. You may subtract the first point from the second or reverse both subtractions; either gives the same slope. Reversing only one subtraction gives the wrong sign. Keep units: if y is cost in pounds and x is quantity in units, the slope is pounds per unit, not a unit-free number.

Slide 5: Intercepts locate the crossings with axes

An intercept is where a graph crosses an axis. In $y = b + mx$, b is the y -intercept because $x = 0$ gives $y = b$. To find the x -intercept, set $y = 0$ and solve. For $y = 10 - 2x$: y -intercept = 10; $0 = 10 - 2x$ gives x -intercept = 5.



The intercepts of $y = 10 - 2x$ are $(0, 10)$ and $(5, 0)$.

Walkthrough

The two intercepts require different substitutions. On the vertical axis the horizontal coordinate is zero, so set x to zero. On the horizontal axis the vertical coordinate is zero, so set y to zero. In our example the crossings are the points zero comma ten and five comma zero. The slope is minus two, meaning a one-unit horizontal increase is associated with a two-unit vertical decrease. An algebraic intercept may lie outside the meaningful domain. A cost line with a positive fixed charge and positive slope, for instance, might have a negative horizontal intercept that does not describe a feasible production quantity.

Slide 6: Positive, negative, zero and undefined slopes

A positive slope rises from left to right; a negative slope falls. A horizontal line has slope zero because y does not change. A vertical line has undefined slope because its horizontal change is zero, which would require division by zero. Visual steepness also depends on axis scales, so compare calculated slopes with their units.

Walkthrough

Do not confuse a negative slope with a negative value. A falling line can remain entirely above the horizontal axis. Similarly, a rising line can include negative vertical values. Slope describes how values change, not where the line sits. A horizontal line has no vertical change, so its slope is zero divided by a non-zero horizontal change. A vertical line has the opposite issue: the horizontal denominator is zero, so its slope is undefined rather than zero. Finally, stretching the drawing vertically can make the same relationship look steeper. Numerical slopes and stated units protect against misleading visual comparisons.

Slide 7: Movement along a line versus a changed line

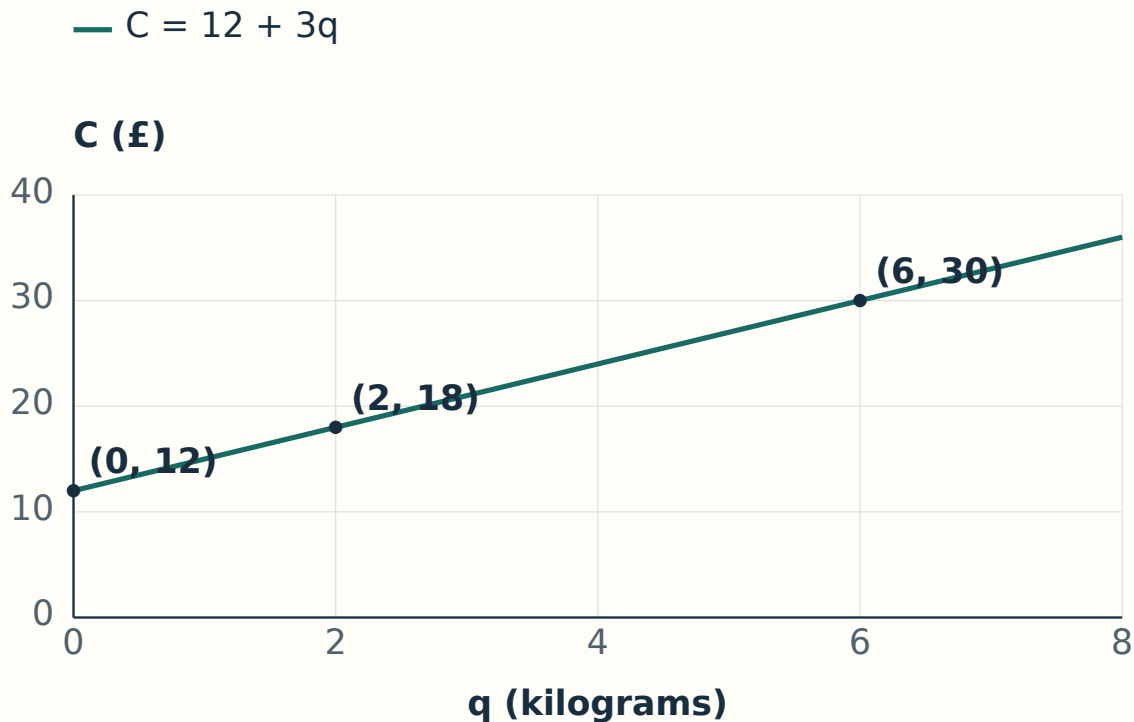
Changing x within an unchanged equation moves the point along the existing line. Changing an equation's parameter changes the relationship itself. In $y = b + mx$, increasing b shifts the line upwards without changing its slope. Changing m changes the slope. Keep these two kinds of change separate when interpreting economic diagrams.

Walkthrough

Consider a cost relationship with a fixed charge and a constant per-unit charge. Ordering more units under the same charges moves us along the existing cost line. Increasing the fixed charge changes the cost associated with every quantity, so the entire line shifts upwards. Increasing the per-unit charge instead changes how quickly cost rises with quantity; the line becomes steeper when the axis scales stay fixed. This distinction is important because an observed point moving upwards does not by itself tell us which mechanism occurred. We need to know whether quantity changed, the underlying terms changed, or both changed at once.

Slide 8: Worked example: plot an illustrative cost line

A fictional delivery service charges $C = 12 + 3q$, where C is pounds and q is kilograms delivered. For $q = 0$: $C = 12 + 3 \times 0 = 12$. For $q = 2$: $C = 12 + 6 = 18$. For $q = 6$: $C = 12 + 18 = 30$. Plot $(0, 12)$, $(2, 18)$, $(6, 30)$.



A £12 fixed charge plus £3 per kilogram produces a straight cost line.

Walkthrough

Draw quantity in kilograms horizontally and total cost in pounds vertically. A horizontal range from zero to six and a vertical range from zero to thirty will include all three points. Mark equal steps on each axis, then locate the calculated pairs. The zero-quantity point represents the fixed charge built into this illustrative formula; whether such a charge would really be payable with no delivery depends on the actual service terms. Here it is part of the stated model. Because kilograms are divisible, the relationship includes intermediate quantities. Join the points with a straight line over the non-negative quantity range being considered.

Slide 9: Worked example: interpret the slope and intercept

Using $(2, 18)$ and $(6, 30)$, slope $= (30 - 18)/(6 - 2) = 12/4 = £3$ per kilogram. The vertical intercept is £12. Algebra gives the horizontal intercept: $0 = 12 + 3q$; $-12 = 3q$; $q = -4$. That negative mass lies outside the delivery model's non-negative domain.

Walkthrough

The slope calculation recovers the coefficient of q , which is a useful check that our plotted values are consistent with the formula. Four additional kilograms raise cost by twelve pounds, so each kilogram adds three pounds. The intercept of twelve pounds is the model's fixed charge, not the price per kilogram. Solving for a horizontal intercept produces minus four kilograms. This is a valid feature of the unrestricted mathematical line, but it is not a possible delivery mass. Draw the meaningful non-negative part as the model, and explain rather than physically interpreting any extension into negative quantities.

Slide 10: Worked example: draw a budget boundary

An illustrative £48 budget buys beans at £6 per kilogram and rice at £4 per kilogram. Let x be kilograms of beans and y kilograms of rice. Affordable combinations satisfy $6x + 4y \leq 48$, with $x \geq 0$ and $y \geq 0$. The full-spending boundary is $6x + 4y = 48$.

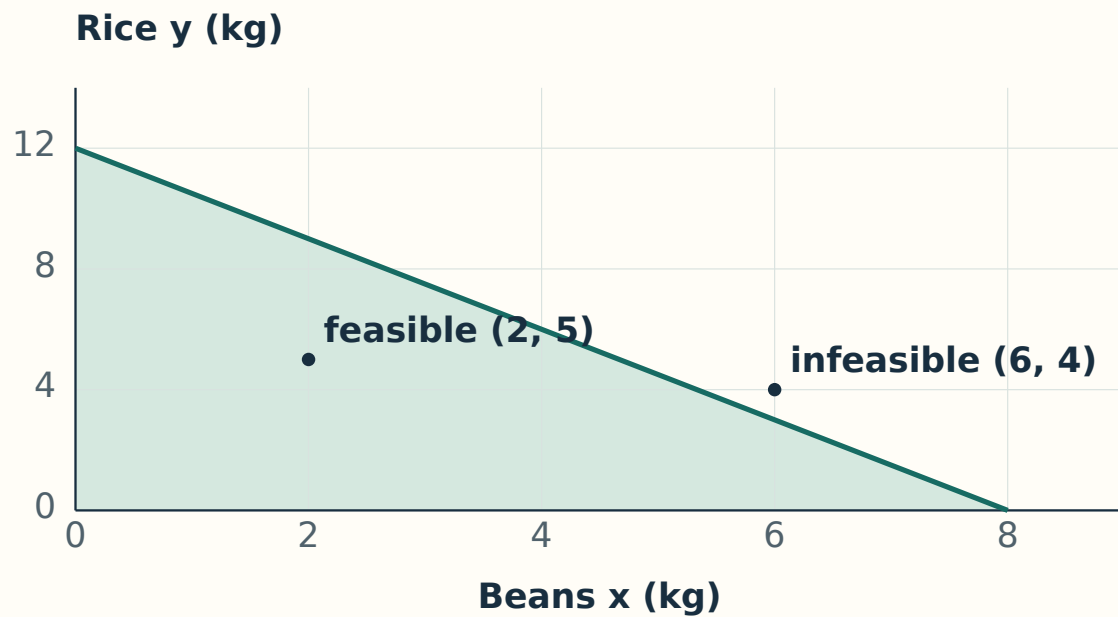
Walkthrough

We are now graphing two goods rather than a quantity and its cost. Put beans on the horizontal axis and rice on the vertical axis. Each combination has a total cost: six pounds times bean kilograms plus four pounds times rice kilograms. The budget allows spending less than or exactly forty-eight pounds, which explains the less-than-or-equal sign. To draw the boundary, temporarily replace that sign with equality. Set y to zero: six x equals forty-eight. Divide both sides by six: $x = 48 \div 6 = 8$. Set x to zero: four y equals forty-eight. Divide both sides by four: $y = 48 \div 4 = 12$. Join the intercepts eight comma zero and zero comma twelve.

Slide 11: Worked example: choose the affordable side

Boundary intercepts are $(8, 0)$ and $(0, 12)$. Test $(2, 5)$: cost = $6 \times 2 + 4 \times 5 = 12 + 20 = £32 \leq £48$, so it is feasible. Test $(6, 4)$: cost = $36 + 16 = £52 > £48$, so it is infeasible. Shade the first-quadrant region on or below the boundary.

$$6x + 4y = 48$$



The shaded region contains all non-negative bundles costing no more than £48.

Walkthrough

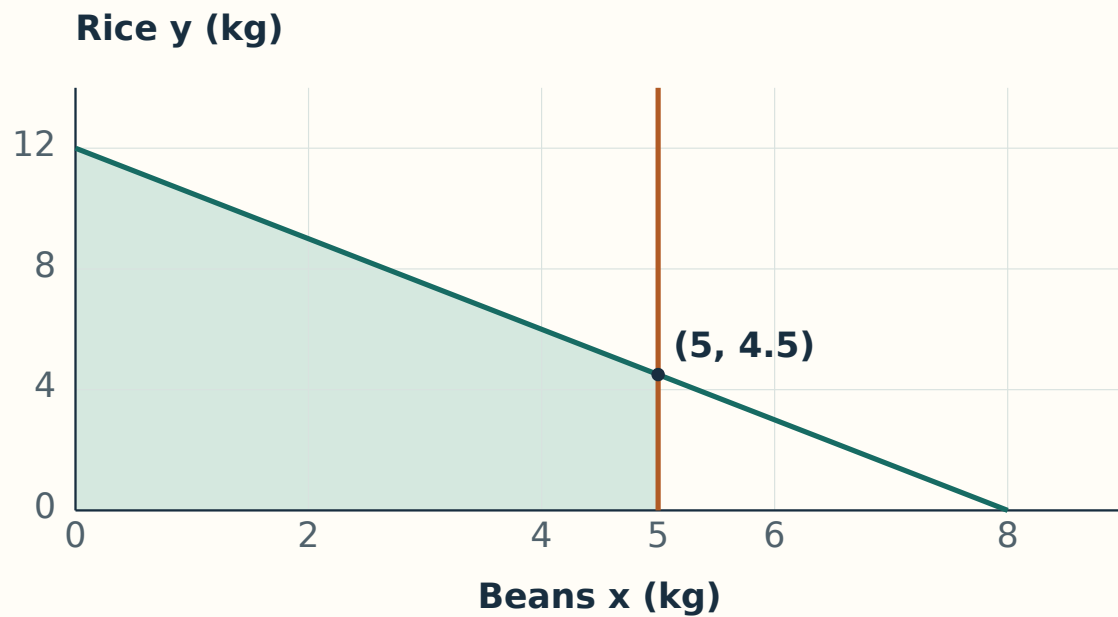
A feasible combination satisfies every stated restriction. The first test point spends thirty-two pounds and leaves sixteen pounds unused, so it lies inside the affordable region. The second spends fifty-two pounds and exceeds the budget by four pounds. The origin is also affordable because buying neither good costs zero in this model. Use a solid boundary because equality is included: combinations spending exactly forty-eight pounds are allowed. Rearranging the boundary gives $4y = 48 - 6x$, then $y = 12 - 1.5x$. Its slope means an extra kilogram of beans requires giving up one point five kilograms of rice when spending remains at the budget limit.

Slide 12: Several constraints mean an intersection

A feasible region must satisfy all constraints together. Add $x \leq 5$ to the previous budget model. At $x = 5$, the budget boundary gives $6 \times 5 + 4y = 48$, so $4y = 18$ and $y = 4.5$. The remaining region has corners $(0, 0)$, $(5, 0)$, $(5, 4.5)$, $(0, 12)$.

$$6x + 4y = 48$$

$$x = 5$$



Intersect the budget region with $x \leq 5$; the new corner is (5, 4.5).

Walkthrough

The additional restriction might represent a five-kilogram availability limit in this invented scenario. Draw the vertical line x equals five and retain only points to its left, including the line itself. Then keep only points that also satisfy the budget and non-negativity conditions. The word “and” is important: satisfying one constraint is insufficient. The four listed corners outline the intersection. Fractional coordinates are acceptable here because kilograms are divisible. With indivisible items, only permitted whole-number points within the region would describe actual choices. A feasible region shows what is allowed; it does not identify a best choice until an objective or preference is specified.

Slide 13: Read a graph without claiming too much

Check axis labels, units, scale, time period and the range shown. Association means quantities vary together; causation means a change in one produces a change in another. A graph alone usually cannot establish causation. A truncated vertical axis can magnify apparent changes, and extending a line beyond observed or modelled values can be unreliable.

Walkthrough

A graph can be arithmetically accurate while encouraging a mistaken impression. Starting the vertical axis near the smallest plotted value can make a modest difference look dramatic, so inspect the tick labels rather than judging height alone. Distinguish a theoretical relationship based on stated assumptions from a scatter of observed values. Observed quantities may move together because a third factor affects both, because influence runs in the opposite direction, or because of selection and chance. Interpolation uses the relationship within its supported range; extrapolation extends beyond that range. Neither a straight line nor a neat pattern guarantees that such an extension will remain valid.

Slide 14: A graph-reading and drawing checklist

Name the variables and units; choose consistent scales; calculate and plot reliable points; identify slope and relevant intercepts; state the domain. For inequalities, draw the boundary and test a point to select the correct side. Combine all restrictions. Finish with a sentence explaining what the picture supports and what it cannot establish.

Walkthrough

Connect this routine to the earlier lessons. Units tell you what each axis means. Algebra supplies the points, intercepts and constraints. Ratios underlie slope, while percentage changes offer a different way to compare movements when the base is known. For an inequality with a strict symbol, use a dashed boundary to show that equality is excluded; for an inclusive symbol, use a solid boundary. A test point should not lie on the boundary, because it must distinguish the two sides. In the exercises, calculate from the stated relationships rather than judging a sketch by eye, and keep any causal conclusion within the evidence actually supplied.

Questions students ask

Why is a coordinate written in a particular order?

The first coordinate identifies the horizontal position and the second the vertical position. Reversing them usually changes the point. Axis labels also determine the meaning and units of each coordinate.

Is the steepest-looking line always the one with the largest slope?

Only when the relevant axis scales and units are comparable. Stretching an axis changes apparent steepness. Calculate vertical change divided by horizontal change and retain its units.

Can a feasible point leave some budget unspent?

Yes, when the constraint says spending must be less than or equal to the budget. Equality describes the boundary, while points with lower spending can also be feasible. Other constraints still need to be satisfied.

Does the boundary itself belong to the feasible region?

It belongs when the inequality includes equality, shown by $\leq$ or $\geq$. A strict $<$ or $>$ excludes the boundary. A solid line indicates inclusion, while a dashed line indicates exclusion.

Why does a feasible region not tell me the best choice?

Feasibility answers whether a combination is allowed. Choosing the best feasible combination requires an additional objective, such as a stated profit formula or the chooser's preferences. Different objectives can select different points.

If two quantities rise together, has one caused the other?

The graph shows an association, but that is not enough to establish causation. Other influences, reverse causation or the way observations were selected may explain the pattern. Additional evidence and a justified method are needed.

Your exercises

Submit your answers on the lesson page at craigwright.net/learning. Numerical and choice checks receive server feedback. Written responses are saved ungraded. No assessment answer key appears in this guide.

1. Calculate a slope from two points

An invented straight cost line contains the points (1, 8) and (5, 24), where the first coordinate is quantity in units and the second is total cost in pounds. Calculate its slope in pounds per unit.

Slope (£/unit)

2. Find both intercepts

Consider $y = 18 - 3x$. Give the x-coordinate where the line crosses the horizontal axis, and the y-coordinate where it crosses the vertical axis. Enter the coordinates requested, not coordinate pairs.

Horizontal-axis crossing: x-coordinate

Vertical-axis crossing: y-coordinate

3. Find a boundary combination

An invented £60 budget can buy good x at £3 per unit and good y at £5 per unit. Both goods have non-negative whole-number quantities. If exactly 5 units of x are bought, what is the maximum affordable quantity of y?

Maximum quantity of y (units)

4. Test three candidate combinations

A fictional feasible region is defined by $4x + 3y \leq 36$, $x \geq 0$ and $y \geq 0$. Classify each point using all three constraints: A = (3, 6), B = (6, 5), C = (9, 0). Points on an included boundary satisfy that constraint.

Point A

1. Feasible
2. Infeasible

Point B

1. Feasible
2. Infeasible

Point C

1. Feasible
2. Infeasible

5. Choose a supported interpretation

Four fictional cafés report these pairs: (outdoor tables, daily drinks sold) = (2, 40), (4, 50), (6, 70), (8, 80). Nothing is reported about café size, location, weather or how cafés were selected. Which conclusion is supported by these observations alone?

Conclusion

1. More outdoor tables are associated with more daily drinks sold in these four observations.
2. Adding outdoor tables has been proved to cause higher drinks sales.
3. Every café gains exactly ten daily drinks sales for each extra table.
4. Every café with eight outdoor tables must sell eighty drinks each day.