

# A graph connects two quantities

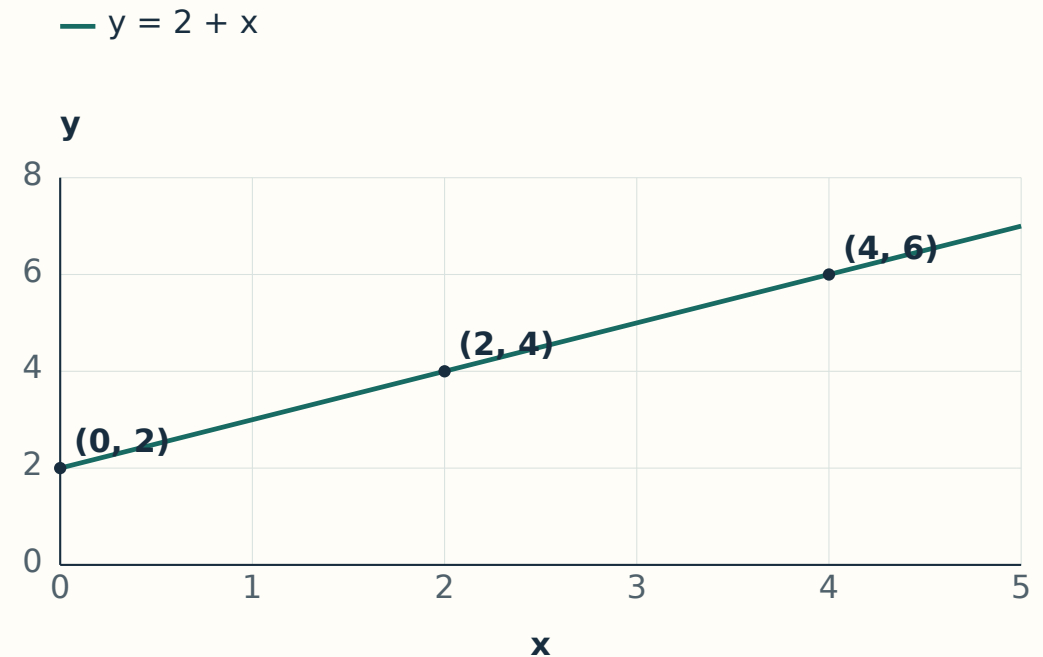
A coordinate graph has a horizontal axis and a vertical axis. Label each axis with its variable and unit. Their crossing is the origin, usually  $(0, 0)$ . A point represents a pair of values. In this lesson, draw simple graphs on paper as you read; the instructions specify the coordinates and boundaries you need.

# Ordered pairs and consistent scales

An ordered pair  $(x, y)$  gives the horizontal value first and the vertical value second. To plot  $(3, 7)$ , move to  $x = 3$ , then to  $y = 7$ . Choose equally spaced tick marks on each axis. The two axes may use different scales, but each individual linear scale must stay consistent.

# Build points from a formula

For  $y = 2 + x$ , choose values of  $x$  and calculate the matching  $y$  values. If  $x = 0$ ,  $y = 2$ ; if  $x = 2$ ,  $y = 4$ ; if  $x = 4$ ,  $y = 6$ . Plot  $(0, 2)$ ,  $(2, 4)$  and  $(4, 6)$ . A linear relationship places these points on one straight line.

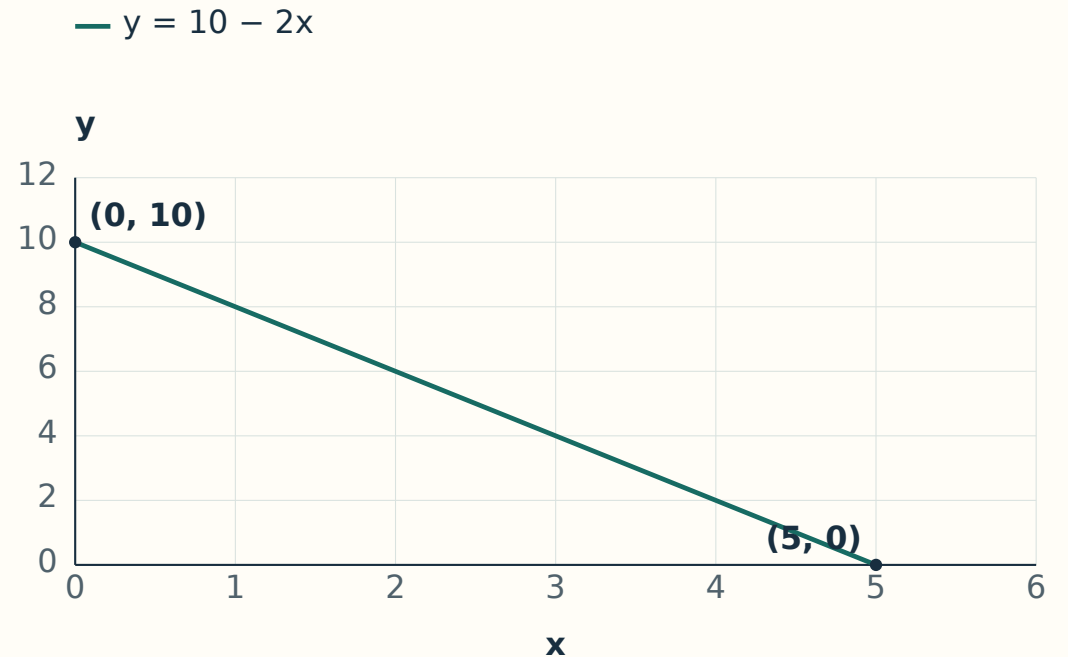


# Slope means vertical change per horizontal change

The slope of a non-vertical straight line is  $m = (y_2 - y_1)/(x_2 - x_1)$ . It measures the change in  $y$  for one unit of  $x$ . Between  $(1, 5)$  and  $(4, 11)$ ,  $m = (11 - 5)/(4 - 1) = 6/3 = 2$ . Subtract coordinates in the same point order.

# Intercepts locate the crossings with axes

An intercept is where a graph crosses an axis. In  $y = b + mx$ ,  $b$  is the y-intercept because  $x = 0$  gives  $y = b$ . To find the x-intercept, set  $y = 0$  and solve. For  $y = 10 - 2x$ :  
y-intercept = 10;  $0 = 10 - 2x$  gives x-intercept = 5.



# Positive, negative, zero and undefined slopes

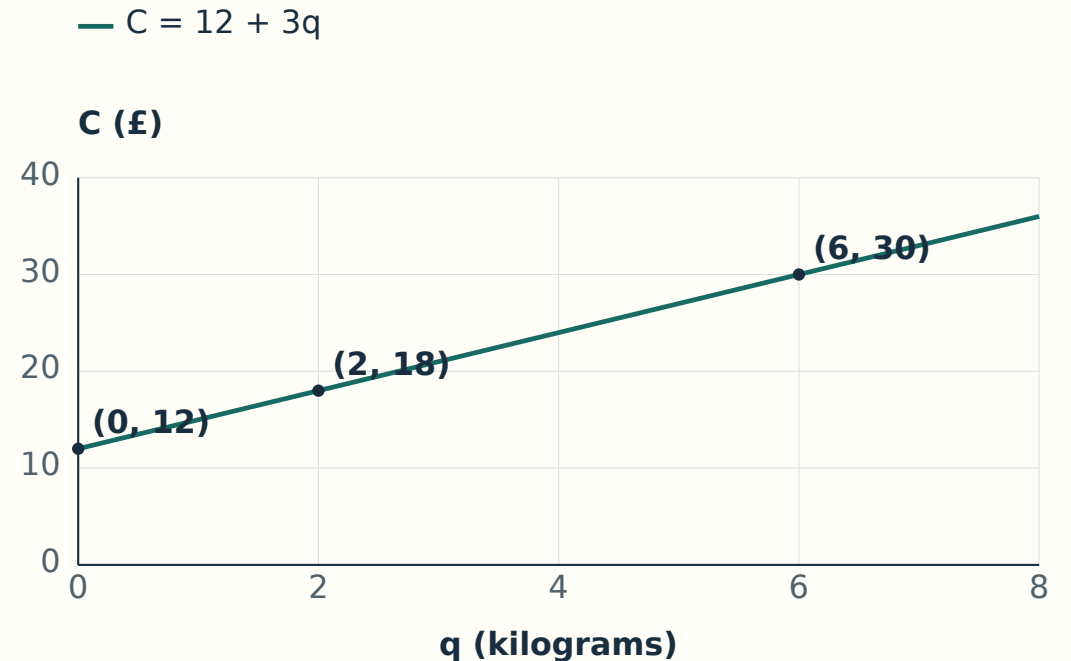
A positive slope rises from left to right; a negative slope falls. A horizontal line has slope zero because  $y$  does not change. A vertical line has undefined slope because its horizontal change is zero, which would require division by zero. Visual steepness also depends on axis scales, so compare calculated slopes with their units.

# Movement along a line versus a changed line

Changing  $x$  within an unchanged equation moves the point along the existing line. Changing an equation's parameter changes the relationship itself. In  $y = b + mx$ , increasing  $b$  shifts the line upwards without changing its slope. Changing  $m$  changes the slope. Keep these two kinds of change separate when interpreting economic diagrams.

# Worked example: plot an illustrative cost line

A fictional delivery service charges  $C$  = 12 + 3 $q$ , where  $C$  is pounds and  $q$  is kilograms delivered. For  $q = 0$ :  $C = 12 + 3 \times 0 = 12$ . For  $q = 2$ :  $C = 12 + 6 = 18$ . For  $q = 6$ :  $C = 12 + 18 = 30$ . Plot (0, 12), (2, 18), (6, 30).



# Worked example: interpret the slope and intercept

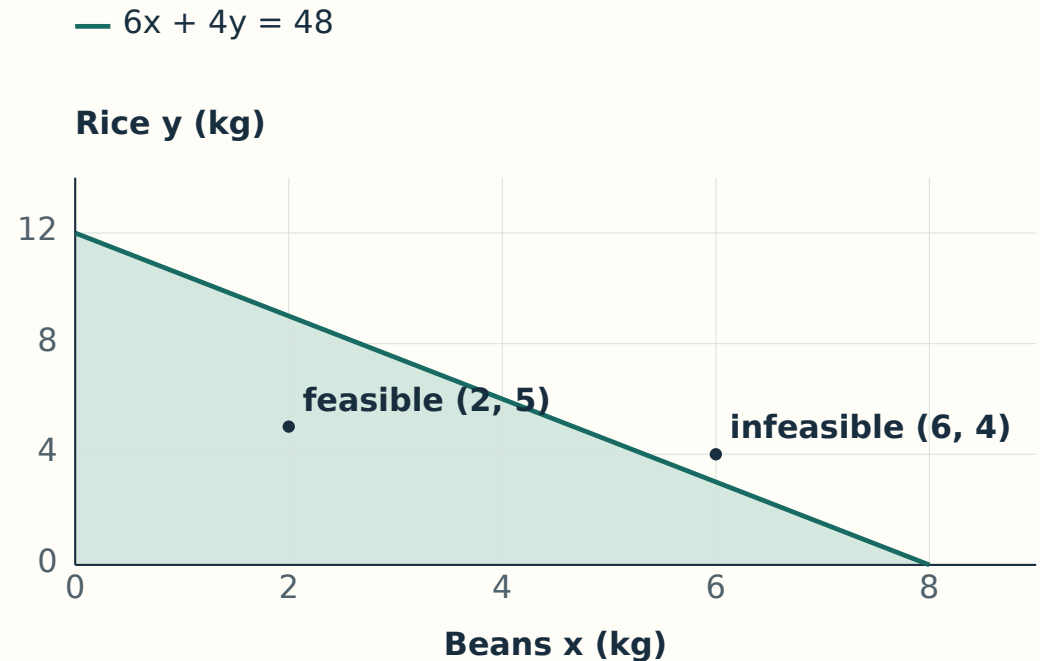
Using (2, 18) and (6, 30), slope =  $(30 - 18)/(6 - 2) = 12/4 = \text{£}3$  per kilogram. The vertical intercept is £12. Algebra gives the horizontal intercept:  $0 = 12 + 3q$ ;  $-12 = 3q$ ;  $q = -4$ . That negative mass lies outside the delivery model's non-negative domain.

## Worked example: draw a budget boundary

An illustrative £48 budget buys beans at £6 per kilogram and rice at £4 per kilogram. Let  $x$  be kilograms of beans and  $y$  kilograms of rice. Affordable combinations satisfy  $6x + 4y \leq 48$ , with  $x \geq 0$  and  $y \geq 0$ . The full-spending boundary is  $6x + 4y = 48$ .

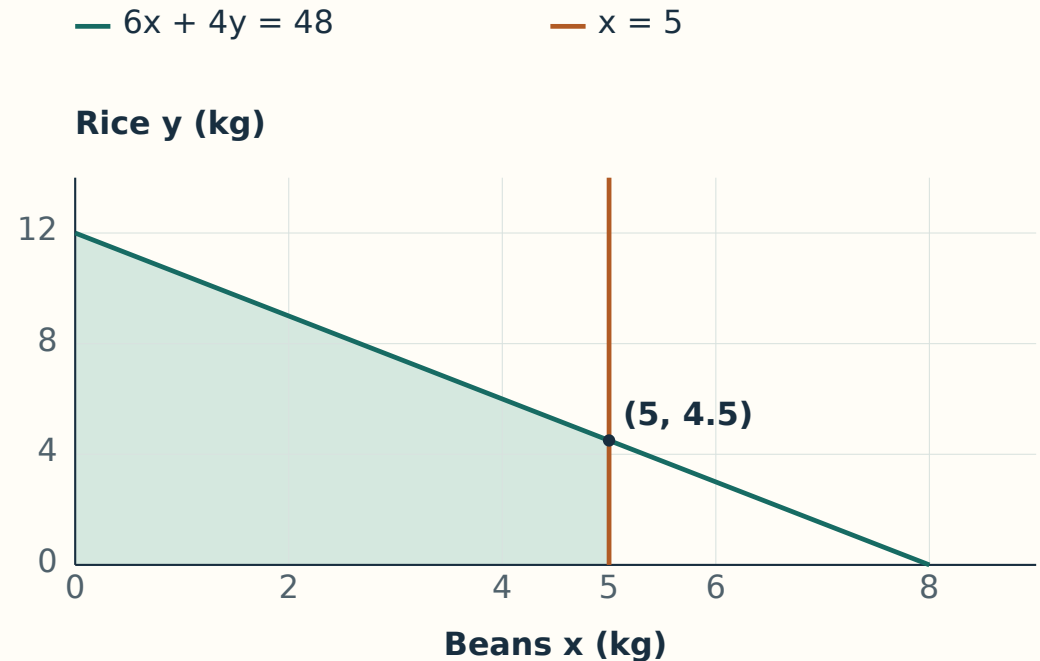
# Worked example: choose the affordable side

Boundary intercepts are (8, 0) and (0, 12). Test (2, 5):  $\text{cost} = 6 \times 2 + 4 \times 5 = 12 + 20 = \text{£}32 \leq \text{£}48$ , so it is feasible. Test (6, 4):  $\text{cost} = 36 + 16 = \text{£}52 > \text{£}48$ , so it is infeasible. Shade the first-quadrant region on or below the boundary.



# Several constraints mean an intersection

A feasible region must satisfy all constraints together. Add  $x \leq 5$  to the previous budget model. At  $x = 5$ , the budget boundary gives  $6 \times 5 + 4y = 48$ , so  $4y = 18$  and  $y = 4.5$ . The remaining region has corners  $(0, 0)$ ,  $(5, 0)$ ,  $(5, 4.5)$ ,  $(0, 12)$ .



# Read a graph without claiming too much

Check axis labels, units, scale, time period and the range shown. Association means quantities vary together; causation means a change in one produces a change in another. A graph alone usually cannot establish causation. A truncated vertical axis can magnify apparent changes, and extending a line beyond observed or modelled values can be unreliable.

# A graph-reading and drawing checklist

Name the variables and units; choose consistent scales; calculate and plot reliable points; identify slope and relevant intercepts; state the domain. For inequalities, draw the boundary and test a point to select the correct side. Combine all restrictions. Finish with a sentence explaining what the picture supports and what it cannot establish.