

Percentage means parts per hundred

A percentage expresses a quantity relative to one hundred equal parts. Thus $25\% = 25/100 = 0.25$. To convert a percentage to a decimal, divide by 100; to convert a decimal to a percentage, multiply by 100. A percentage alone does not identify the size of the underlying whole.

Find a percentage of a quantity

To find $p\%$ of an amount A , calculate $A \times p/100$. For example, 12% of £250
 $= 250 \times 12/100 = 250 \times 0.12 = £30$. Here £250 is the base: the whole
against which the percentage is measured. Label the base before doing the
multiplication.

Multipliers include the amount that remains

A multiplier is the factor used to obtain a new value. Increasing A by $p\%$ gives $A \times (1 + p/100)$. Decreasing A by $p\%$ gives $A \times (1 - p/100)$. A 20% increase uses 1.20; a 20% decrease uses 0.80. Multiplying by 0.20 finds only the change amount.

Percentage change uses the starting value

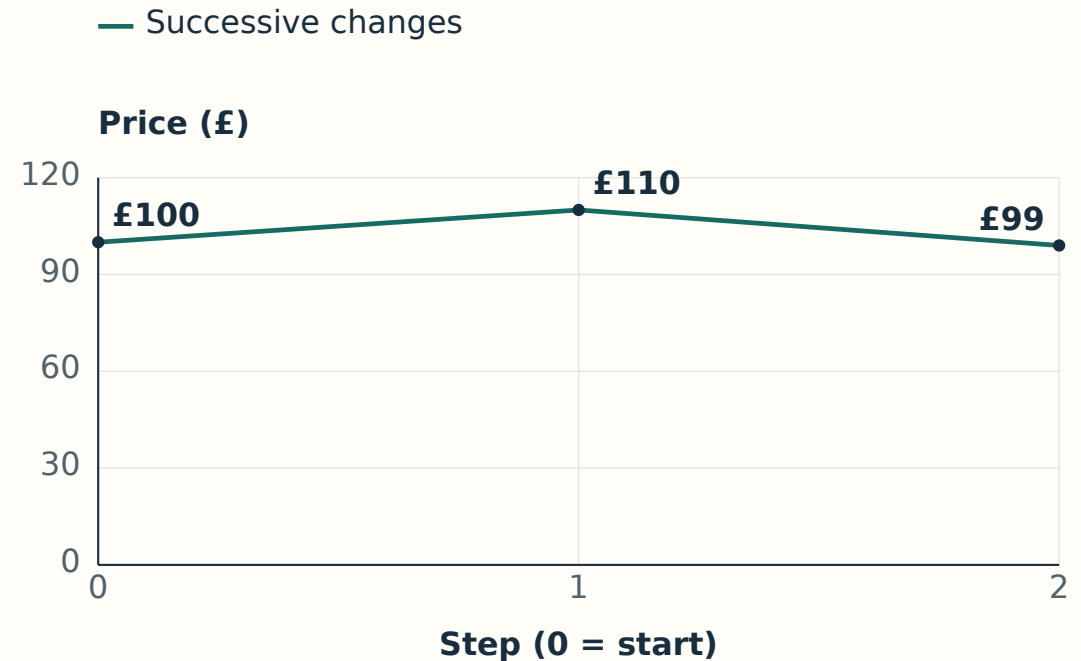
Percentage change = $[(\text{new value} - \text{original value}) \div \text{original value}] \times 100$.
If an illustrative quantity rises from 40 to 50, the change is 10 and the percentage change is $10 \div 40 \times 100 = 25\%$. A negative result indicates a decrease. The usual formula is undefined when the original value is zero.

Percentage points compare percentage levels

A percentage point is one unit on a percentage scale. If a fictional participation rate rises from 30% to 36%, it rises by $36 - 30 = 6$ percentage points. Relative to its original level, it rises by $(36 - 30) \div 30 \times 100 = 20\%$. These are different descriptions of the same movement.

Opposite percentages do not usually cancel

An invented price of £100 rises by 10% to £110. A subsequent 10% decrease removes £11, leaving £99. The second percentage uses a different base. The combined multiplier is $1.10 \times 0.90 = 0.99$, so the net movement is a 1% decrease, even though the stated percentages have equal sizes.



Compounding builds on earlier changes

Compounding means applying each percentage change to the amount reached after the previous change. At a constant decimal rate r for n periods, final amount = initial amount $\times (1 + r)^n$. The exponent n means repeated multiplication. With varying rates, multiply the separate period multipliers in sequence.

Worked example: discount an illustrative price

A fictional item lists at £120. A 15% discount is applied first, followed by a 5% charge on the discounted price. Discount amount = $£120 \times 15/100 = £18$. Discounted price = $£120 - £18 = £102$. Equivalently, $£120 \times 0.85 = £102$. The next calculation must use £102 as its base.

Worked example: apply the charge and compare

Charge = $\pounds 102 \times 0.05 = \pounds 5.10$. Final price = $\pounds 102 + \pounds 5.10 = \pounds 107.10$.

Combined multiplier = $0.85 \times 1.05 = 0.8925$. Net change = $(107.10 - 120) \div 120 \times 100 = -12.90 \div 120 \times 100 = -10.75\%$. The final price is 10.75% below the original price.

Worked example: set up repeated growth

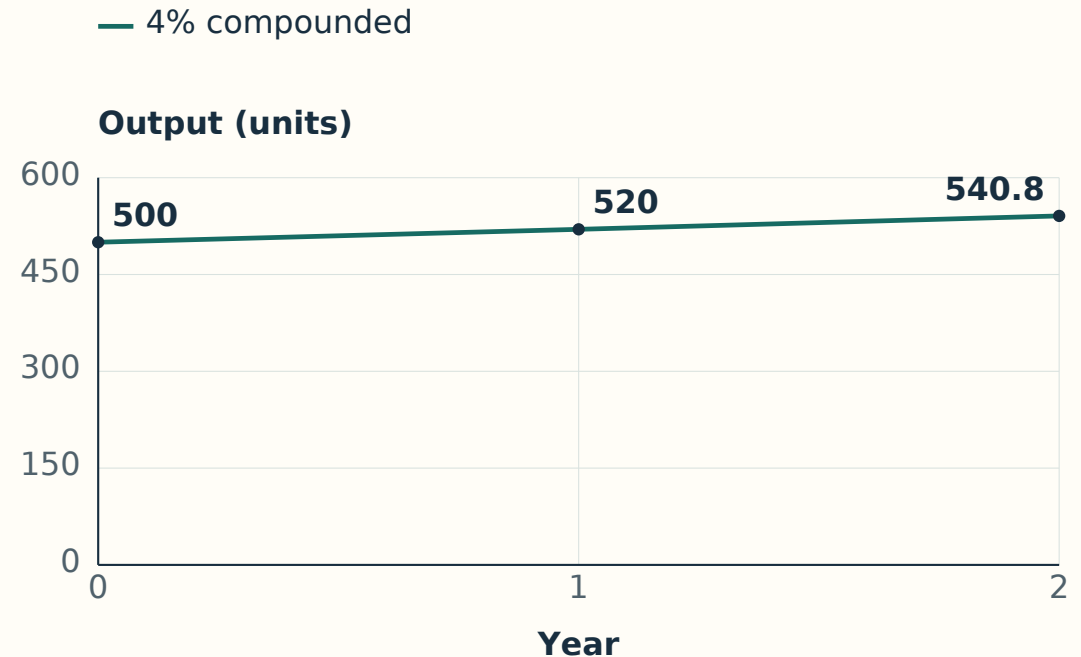
An invented planning model starts with 500 units of output. Output grows by 4% in each of two years, with each increase based on the previous year's output. The annual multiplier is $1 + 4/100 = 1.04$. After year one: $500 \times 1.04 = 520$ units. The second year starts from 520.

Worked example: finish and measure total growth

Year-two increase = $520 \times 0.04 = 20.8$ units. Final output = $520 + 20.8 = 540.8$ units. Equivalently, $500 \times 1.04^2 = 500 \times 1.0816 = 540.8$.

Overall percentage growth = $(540.8 - 500) \div 500 \times 100 = 8.16\%$.

Adding $4\% + 4\%$ would miss growth on the earlier increase.



Reverse a percentage change by dividing

If a value after a 15% increase is £69, the original value satisfies $\text{original} \times 1.15 = 69$. Therefore $\text{original} = 69 \div 1.15 = \text{£}60$. Subtracting 15% of £69 would use the wrong base. For a known discount, divide the final value by the fraction of the original that remains.

Report the base, period and rounding

A useful percentage statement identifies its base, its time period and whether the change is relative or measured in percentage points. Round only the final answer unless instructed otherwise. A large percentage increase can describe a small absolute change, so report the underlying amounts when they matter. Percentages alone cannot establish why a change occurred.

A reliable percentage decision tree

Ask what the question wants. A share requires $\text{base} \times \text{decimal percentage}$. A new amount requires a multiplier. A percentage change requires $\text{change} \div \text{original} \times 100$. A change between percentage levels may require percentage points. Successive changes require multiplied factors. Recovering an original value requires dividing by the known multiplier.