

A number needs a meaning

A quantity combines a number with a meaning: £24 is money, 24 kilograms is mass, and 24 hours is time. A unit is the standard used to measure that quantity. In this lesson, every answer should say what is being counted or measured. Our numerical scenarios are invented for learning.

Totals and rates answer different questions

A total describes an amount, such as £48 spent. A rate compares quantities with different units, such as £12 per hour. The word “per” means “for each” and usually signals division. Multiplying a rate by the relevant quantity produces a total: $(£12 \text{ per hour}) \times 4 \text{ hours} = £48$.

Convert units before combining quantities

A conversion changes the numerical description, not the physical amount. One kilogram equals 1,000 grams, so $300 \text{ grams} = 300 \div 1,000 = 0.3$ kilograms. One hour equals 60 minutes. Before adding quantities or comparing prices, express the relevant measurements in matching units. Never add kilograms directly to grams.

Fractions describe equal parts

A fraction a/b means a equal parts when the whole has been divided into b equal parts. The numerator a counts the parts; the denominator b sets their size. For $3/4$ of a £20 amount, divide £20 into four equal parts, then take three: $£20 \div 4 \times 3 = £15$.

Equivalent fractions and decimal notation

Equivalent fractions name the same share. Multiplying or dividing both numerator and denominator by the same non-zero number preserves the value: $6/8 = 3/4$. To write a fraction as a decimal, divide the numerator by the denominator: $3 \div 4 = 0.75$. A terminating decimal ends; some decimal representations continue indefinitely.

Calculate with fractions carefully

To add fractions, first use a common denominator: $1/2 + 1/3 = 3/6 + 2/6 = 5/6$. To multiply, multiply numerators and denominators: $(2/3) \times (3/5) = 6/15 = 2/5$. To divide by a non-zero fraction, multiply by its reciprocal, which swaps its numerator and denominator.

Ratios compare; rates include units

A ratio compares relative amounts. A red-to-blue ratio of 2:3 means two red items for every three blue items. There are five parts altogether, so red items form $\frac{2}{5}$ of the total. A unit rate gives an amount for one unit, such as pounds per kilogram, and helps compare differently sized purchases.

Worked example: plan a unit-price calculation

An invented flour pack contains 750 grams and costs £1.80. We want its price per kilogram and the cost of 4.5 kilograms at the same rate. First convert the mass: $750 \div 1,000 = 0.75$ kilograms. Then set up the rate: price per kilogram = $\text{£}1.80 \div 0.75$ kilograms.

Worked example: calculate and verify the cost

Price per kilogram = $1.80 \div 0.75 = 180 \div 75 = \text{£}2.40$ per kilogram. Cost of 4.5 kilograms = $2.40 \times 4.5 = 2.40 \times (4 + 0.5) = 9.60 + 1.20 = \text{£}10.80$.

Check another way: $4.5 \div 0.75 = 6$ packs; $6 \times \text{£}1.80 = \text{£}10.80$.

Worked example: divide a budget in a ratio

An illustrative £360 budget is divided between rent, food and travel in the ratio 5:3:1. The order matters: rent receives five parts, food three and travel one. Total parts = $5 + 3 + 1 = 9$. Value of one part = $£360 \div 9 = £40$.

Worked example: allocate and check the whole

Rent = $5 \times £40 = £200$. Food = $3 \times £40 = £120$. Travel = $1 \times £40 = £40$.

Check the total: $£200 + £120 + £40 = £360$. Food's fraction of the whole is $3/9 = 1/3$; this agrees with $£120 \div £360 = 1/3$.

Direct proportion: scale the same relationship

Two quantities are directly proportional when multiplying one by a factor multiplies the other by the same factor. If four identical tickets cost £18 with no fixed fee, one costs $£18 \div 4 = £4.50$. Seven therefore cost $7 \times £4.50 = £31.50$. The constant unit rate is the key assumption.

Use estimates and sensible precision

An estimate is a deliberately approximate calculation used to check size. Rounding £2.98 to about £3 makes ten items cost about £30. Precision should fit the question: ordinary money totals are usually stated to two decimal places. Keep intermediate values unrounded where possible, and state when an answer is approximate.

Your quantity-checking routine

Use five steps: identify the requested quantity; write the starting units; convert to compatible units; calculate; then check the result against the original situation. Ask whether a fraction refers to the right whole and whether a ratio keeps its stated order. A complete answer combines a value, a unit and a short interpretation.