

Quantities, Units, Fractions and Ratios

Build a reliable language for quantities: attach units, convert measurements, calculate with fractions and divide totals in a stated ratio.

Written walkthrough. Work through each explanation alongside the PDF slides. The lesson page also includes a captioned video with a generic synthetic narrator. It is not a recording of Craig Wright.

Fictional examples teach reasoning rather than describe a real business. No external academic credit is awarded.

Learning objectives

1. Attach appropriate units to quantities and distinguish totals from rates.
2. Convert between related units without changing the underlying amount.
3. Simplify fractions and calculate with a common denominator.
4. Use ratios to allocate a total and use unit rates to compare or scale quantities.

Slide 1: A number needs a meaning

A quantity combines a number with a meaning: £24 is money, 24 kilograms is mass, and 24 hours is time. A unit is the standard used to measure that quantity. In this lesson, every answer should say what is being counted or measured. Our numerical scenarios are invented for learning.

Walkthrough

Begin by separating the calculation from the interpretation. The calculation 12×2 gives 24, but it cannot tell us whether that answer describes pounds, kilograms or hours. We need the question for that. Write the unit beside each starting number, and carry it through to your answer. This small habit prevents many economic mistakes because prices, wages, output and time are different kinds of quantities. You do not need to rush. For each example, first explain in ordinary words what the answer is supposed to describe, then calculate it.

Slide 2: Totals and rates answer different questions

A total describes an amount, such as £48 spent. A rate compares quantities with different units, such as £12 per hour. The word “per” means “for each” and usually signals division. Multiplying a rate by the relevant quantity produces a total: $(£12 \text{ per hour}) \times 4 \text{ hours} = £48$.

Walkthrough

Imagine being told that someone earns twelve pounds. Without a time period, we do not know much about their pay. Twelve pounds per hour gives a rate; forty-eight pounds for four hours gives a total. Notice how the hours in the rate and the hours worked fit together. If we divide the forty-eight-pound total by four hours, we recover twelve pounds per hour. Write rates as a fraction if that helps: pounds divided by hours. Keep the denominator visible, because pounds per hour and hours per pound describe different questions.

Slide 3: Convert units before combining quantities

A conversion changes the numerical description, not the physical amount. One kilogram equals 1,000 grams, so $300 \text{ grams} = 300 \div 1,000 = 0.3 \text{ kilograms}$. One hour equals 60 minutes. Before adding quantities or comparing prices, express the relevant measurements in matching units. Never add kilograms directly to grams.

Walkthrough

Think of conversion as choosing a different-sized measuring unit. A kilogram is a larger unit than a gram, so the same amount needs fewer kilograms than grams. That is why we divide the gram figure by one thousand. Going back from kilograms to grams requires multiplication by one thousand. A quick size check is useful: three hundred grams should be less than one kilogram, so an answer of three hundred thousand kilograms cannot be right. Time needs special care because an hour contains sixty minutes, rather than one hundred. Always write the conversion relationship first.

Slide 4: Fractions describe equal parts

A fraction a/b means a equal parts when the whole has been divided into b equal parts. The numerator a counts the parts; the denominator b sets their size. For $3/4$ of a £20 amount, divide £20 into four equal parts, then take three: $£20 \div 4 \times 3 = £15$.

Walkthrough

The denominator is not just another number to manipulate. It tells you what one part means. If a twenty-pound amount is split into four equal parts, each part is five pounds. Taking three of those parts gives fifteen pounds. The parts must be equal for this interpretation to work. A fraction can also be larger than one: five quarters means five pieces of size one quarter, or one whole and one quarter. When using a fraction in an economic question, identify the whole explicitly. Three quarters of income and three quarters of spending may be different amounts.

Slide 5: Equivalent fractions and decimal notation

Equivalent fractions name the same share. Multiplying or dividing both numerator and denominator by the same non-zero number preserves the value: $6/8 = 3/4$. To write a fraction as a decimal, divide the numerator by the denominator: $3 \div 4 = 0.75$. A terminating decimal ends; some decimal representations continue indefinitely.

Walkthrough

Picture six shaded pieces in a whole cut into eight equal pieces. Pair the small pieces together and you have three shaded pairs out of four pairs. The shaded share has not changed, even though its name has. This is why simplifying requires the same operation on the numerator and denominator. Dividing only the numerator would change the fraction. Decimal notation gives another useful name for the same value. For example, one third is approximately zero point three three three, but rounding it to two decimal places introduces a small difference. Keep extra digits during a calculation and round at the end.

Slide 6: Calculate with fractions carefully

To add fractions, first use a common denominator: $1/2 + 1/3 = 3/6 + 2/6 = 5/6$. To multiply, multiply numerators and denominators: $(2/3) \times (3/5) = 6/15 = 2/5$. To divide by a non-zero fraction, multiply by its reciprocal, which swaps its numerator and denominator.

Walkthrough

Addition combines pieces of the same size, so halves and thirds must first be renamed as sixths. Do not add denominators: doing that would change the size of the pieces while you are counting them. Multiplication answers a different question, such as finding two thirds of three fifths of a whole. Division asks how many of one quantity fit into another. For example, one half divided by one quarter equals one half multiplied by four, which equals two. Two quarters fit inside one half. You cannot divide by zero because there is no meaningful finite answer to that operation.

Slide 7: Ratios compare; rates include units

A ratio compares relative amounts. A red-to-blue ratio of 2:3 means two red items for every three blue items. There are five parts altogether, so red items form $2/5$ of the total. A unit rate gives an amount for one unit, such as pounds per kilogram, and helps compare differently sized purchases.

Walkthrough

Read a ratio in the stated order. Two to three for red to blue does not mean that two thirds of all the items are red. The three counts blue parts, not total parts. Add the two and three to find five parts in the combined whole. A rate also compares quantities, but usually retains different measurement units. If a purchase costs money and has a mass, dividing cost by mass produces pounds per kilogram. That gives a common basis for comparison. Ratios alone show relative size; an actual total or another known quantity is needed to recover absolute amounts.

Slide 8: Worked example: plan a unit-price calculation

An invented flour pack contains 750 grams and costs £1.80. We want its price per kilogram and the cost of 4.5 kilograms at the same rate. First convert the mass: $750 \div 1,000 = 0.75$ kilograms. Then set up the rate: price per kilogram = $£1.80 \div 0.75$ kilograms.

Walkthrough

We start by naming both requested answers. The first must be in pounds per kilogram; the second must be in pounds. The packet mass is given in grams, so it does not yet match the kilogram denominator we want. Divide seven hundred and fifty by one thousand to obtain zero point seven five kilograms. Now divide the packet price by that mass. Do not multiply the price by zero point seven five, because that would not find a price for one kilogram. Pause before calculating and predict that a full kilogram should cost more than this three-quarter-kilogram packet.

Slide 9: Worked example: calculate and verify the cost

Price per kilogram = $1.80 \div 0.75 = 180 \div 75 = \text{£}2.40$ per kilogram. Cost of 4.5 kilograms = $2.40 \times 4.5 = 2.40 \times (4 + 0.5) = 9.60 + 1.20 = \text{£}10.80$. Check another way: $4.5 \div 0.75 = 6$ packs; $6 \times \text{£}1.80 = \text{£}10.80$.

Walkthrough

Multiplying both numbers in the division by one hundred removes the decimal places without changing the quotient. One hundred and eighty divided by seventy-five is two point four, giving a rate of two pounds forty per kilogram. We then multiply that rate by four and a half kilograms. Splitting four and a half into four plus one half makes the arithmetic visible. The independent pack-count check reaches the same total. This example assumes the same price rate applies throughout, with no discounts or extra charges. Six whole packs are available here, so the requested mass is also achievable using this packaging.

Slide 10: Worked example: divide a budget in a ratio

An illustrative £360 budget is divided between rent, food and travel in the ratio 5:3:1. The order matters: rent receives five parts, food three and travel one. Total parts = $5 + 3 + 1 = 9$. Value of one part = $\text{£}360 \div 9 = \text{£}40$.

Walkthrough

A ratio gives instructions for relative shares, rather than fixed pound amounts. Our first task is to discover what one ratio part is worth. Add all the parts because the complete budget must cover all three categories. Nine equal parts share three hundred and sixty pounds, so each part is forty pounds. At this stage, do not assign forty pounds to every category. Rent is entitled to five of these equal parts, while travel gets just one. Writing the category names directly above the ratio numbers is a simple way to preserve the correct ordering throughout the calculation.

Slide 11: Worked example: allocate and check the whole

Rent = $5 \times \text{£}40 = \text{£}200$. Food = $3 \times \text{£}40 = \text{£}120$. Travel = $1 \times \text{£}40 = \text{£}40$. Check the total: $\text{£}200 + \text{£}120 + \text{£}40 = \text{£}360$. Food's fraction of the whole is $3/9 = 1/3$; this agrees with $\text{£}120 \div \text{£}360 = 1/3$.

Walkthrough

We have now translated relative shares into actual money. Two checks are helpful. First, add the three allocations to confirm that the whole budget has been assigned exactly once. Second, compare the allocations with the original ratio. Two hundred to one hundred and twenty to forty simplifies, after dividing each amount by forty, to five to three to one. The food share also illustrates the difference between a ratio comparison and a fraction of the whole. Food to rent is three to five, but food as a share of the complete budget is three out of nine parts.

Slide 12: Direct proportion: scale the same relationship

Two quantities are directly proportional when multiplying one by a factor multiplies the other by the same factor. If four identical tickets cost £18 with no fixed fee, one costs $£18 \div 4 = £4.50$. Seven therefore cost $7 \times £4.50 = £31.50$. The constant unit rate is the key assumption.

Walkthrough

A unit-rate method often feels clearer than a memorised cross-multiplication rule. Reduce the known bundle to one item, then build the new bundle. Four tickets to seven tickets is a multiplication by seven quarters, so multiplying eighteen pounds by seven quarters would give the same result. But the method depends on a constant relationship. A booking fee, bulk discount or minimum charge would change the structure. In economic questions, read the words before assuming proportionality. If there is a fixed fee, total cost might still rise with quantity, but it will not necessarily rise in exactly the same proportion.

Slide 13: Use estimates and sensible precision

An estimate is a deliberately approximate calculation used to check size. Rounding £2.98 to about £3 makes ten items cost about £30. Precision should fit the question: ordinary money totals are usually stated to two decimal places. Keep intermediate values unrounded where possible, and state when an answer is approximate.

Walkthrough

Calculators can produce many digits, but extra digits do not automatically make an answer more meaningful. A purchase total expressed in pounds normally ends at pennies, while a unit price might need extra decimal places for a fair comparison. Estimate before relying on the exact calculation. If ten items cost just under three pounds each, a result near three hundred pounds is a warning. Also distinguish rounding from changing the underlying assumptions. Rounding a price slightly is an approximation; assuming a discount exists is a different model. Explain any assumption that materially affects what your answer means.

Slide 14: Your quantity-checking routine

Use five steps: identify the requested quantity; write the starting units; convert to compatible units; calculate; then check the result against the original situation. Ask whether a fraction refers to the right whole and whether a ratio keeps its stated order. A complete answer combines a value, a unit and a short interpretation.

Walkthrough

You now have a routine that will carry into every later lesson. Before pressing calculator buttons, decide what the answer should measure. During the calculation, keep units and category labels visible. Afterwards, check the total, reverse the operation, or estimate the expected size. These checks are complementary: a tidy arithmetic answer can still use the wrong whole or the wrong unit. In the exercises, show yourself the working on paper even when the submission box asks only for a number. Enter numerical answers without currency symbols where the field already supplies the unit, and use the precision requested by the question.

Questions students ask

Why can I not add $\frac{1}{2}$ and $\frac{1}{3}$ by adding their numerators and denominators?

The denominators describe different-sized pieces. Rename the fractions as $\frac{3}{6}$ and $\frac{2}{6}$ so the pieces match; then count five sixths. Adding denominators would change the piece size without justification.

Does a ratio of 2:3 mean that the first category is $\frac{2}{3}$ of the total?

No. It means the first category is two thirds as large as the second category. The combined total has five equal ratio parts, so the first category is two fifths of that total.

How do I know whether to multiply or divide when converting units?

Write a known relationship first, such as 1 kilogram = 1,000 grams. Moving to the larger unit means fewer units for the same amount, so divide. Check by converting your answer back.

Can I always scale a price using the unit-rate method?

Only when the same rate applies to the quantity in question. Fixed fees, discounts, packaging restrictions and additional charges can change the relationship. State the constant-rate assumption when the scenario uses it.

Your exercises

Submit your answers on the lesson page at craigwright.net/learning. Numerical and choice checks receive server feedback. Written responses are saved ungraded. No assessment answer key appears in this guide.

1. Convert a parcel mass

An invented parcel has a mass of 1,350 grams. Express its mass in kilograms. Enter a decimal number.

Mass (kg)

2. Find a share of a materials fund

A practice project has a £280 materials fund. Exactly $\frac{3}{7}$ of the fund is assigned to timber. How many pounds are assigned to timber?

Timber allocation (£)

3. Allocate three departments a total

An illustrative £420 equipment fund is divided between design, assembly and testing in the ratio 2:4:1, in that order. Enter each allocation in pounds.

Design (£)

Assembly (£)

Testing (£)

4. Compare a different pack size

A fictional 600-gram bag of rice costs £1.74. Find the price per kilogram. Then find the cost of buying 3 kilograms at the same rate, with no additional fees or discounts. Give both money answers to two decimal places.

Price per kilogram (£/kg)

Cost of 3 kilograms (£)

5. Combine compatible shares

Two non-overlapping stages use $\frac{1}{4}$ and $\frac{2}{5}$ of the same material stock. Express the combined share as a fraction in its simplest form. Enter a positive integer numerator and a positive integer denominator.

Numerator

Denominator